AVO INVERSION USING LEVENBERG-MARQUARDT OPTIMIZATION TECHNIQUE

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ABSTRACT

AVO is not only well known as gas indicator over the last two decades but also more importantly, AVO provides us with a means for extracting petrophysical parameters from seismic data. Using AVO anomaly one can derive important petrophysical parameters such as Poisson's ratio and S-wave velocity. By knowing S-wave velocity nearly all other petrophysical parameters can be calculated. An effective procedure for inverting AVO anomaly is presented in this paper. It avoids inefficient trial and error steps during the matching process between AVO anomaly and calculated AVO. This method uses Levenberg-Marquardt optimization technique.

Key words*: AVO anomaly, AVO inversion, petrophysical parameters, Levenberg-Marquardt optimization technique*

I. INTRODUCTION

AVO (Amplitude versus Offset) has been widely known in oil and gas industries over the last twenty years. It is a phenomenon which is exploited to extract petrophysical properties from seismic data. Using AVO anomaly one can predict the shear wave velocity from ordinary seismic data. This extracted shear wave velocity in turn can be used to derive other petrophysical parameters such as Poisson's ratio, incompressibility, Young modulus, mu-rho and lambda-rho, etc.

In 1995, The Society of Exploration Geophysicsts has published a comprehensive book entitled "Offset-dependent reflectivity-theory and practice of AVO analysis" edited by Castagna and Backus. This book has become resource information for the industries in searching gas deposits in the subsurface. AVO then is well-known as a good gas indicator which is more powerful than the previous phenomena known as "bright spot" introduced in the seventies.

The success and development of AVO implementation for oil and gas industries is firmly demonstrated with the publication of a book written by Avseth, Mukerji and Mavco (2006) entitled "Quantitative Seismic Interpretation: Applying Rock Physics Tool to Reduce Interpretation Risk". In this book it is demonstrated that AVO analysis can provide significant means for differentiating reservoir rock (lithology) as well as its fluid content.

AVO anomaly will be shortly reviewed in the next paragraph but the most important thing is its inversion. Inversion here means by knowing AVO anomaly one wants to estimate the petrophysical properties which causes the anomaly. The aim of this paper is to demonstrate a specific method for estimating the petrophysical parameters from a given AVO anomaly using the Marquardt optimization technique.

II. THEORETICAL BACKGROUND

The basic theoretical concept of AVO origins from the work by Zoeppritz (1919) who formulated the energy partition as function of angle of incidence when a seismic wave impinges an interface. Zoeppritz formulation is the exact solution of this problem, it formed a 4 x 4 simultaneous equations. Computationally it is complicated, that is why it took a very long time to go before its implementation can be used by industries. We do not intend to rewrite the exact Zoeppritz equation here, but instead we will display an example of a

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specific case represented by curves computed from Zoeppritz's equation (see Figure 1) and discuss its implications.

Figure 1 is the theoretical curve of the AVO phenomena in which P wave incident into an interface with low velocity layer above and higher velocity layer below it. It can be seen that most of the energy is transmitted into the second layer. The reflected energy although is only a small portion has a specific behavior, departing from zero degree angle of incidence we can see that the decrease of the P wave energy (r_{pp}) is compensated by the increase of S wave energy (r_{ps}). There is a discontinuity in this curve which located around 38° (angle of incidence). This discontinuity signifies the existence of a critical angle.

The AVO phenomena can also be observed in a seismic record (CDP gather) as illustrated in Figure 2.

The use of Zoeppritz concept started gaining popularity after his formulation can be simplified by several researchers such as Koefoed (1955), Aki and Richards (1980), Ostrander (1984), Shuey (1985). Using the simplified Zoeppritz's equation the phenomena of reflection and transmission coefficient from a single interface as a function of angle of incidence can be computed easily which makes it favorable for multiple applications purposes such as required in the inversion process. It should be noted that Zoeppritz simplified equation is only valid for angle of incidence up to critical angle, while the complete Zoeppritz equation valid from 0° to 90° (angle of incidence).

III. AVO INVERSION

In geophysics one usually observes an anomaly because of "something" that exists in the subsurface layer. Inversion means: based on the observed anomaly one wants to know the "something" which causes the anomaly.

Basically the AVO inversion consists of fitting the AVO anomaly using a computed curve calculated from a predicted model. When this matching does not fit, one modifies the model's parameters then calculates again

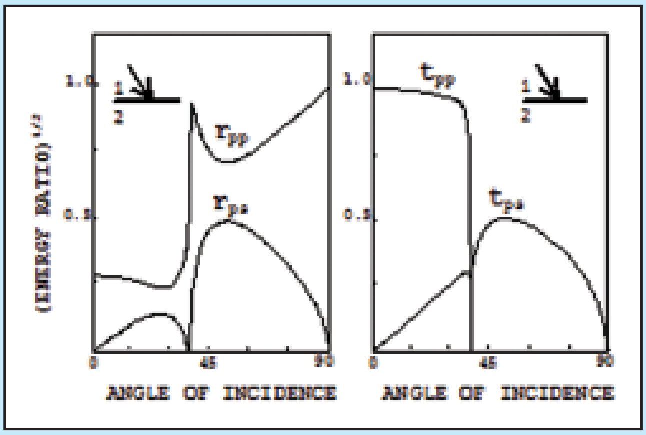

Figure 1
Energy ratio as a function of angle of incidence computed using exact Zoeppritz's equation with a specific model represented by Vp, Vs and rho. (Munadi, 1985)

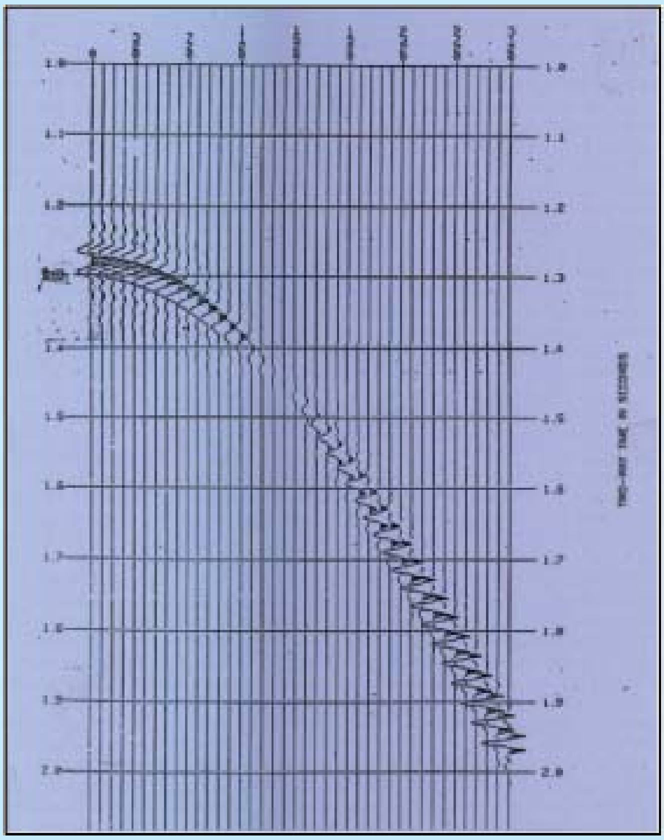

Figure 2
AVO phenomena observed in a noise-free synthetic CDP gather where shale overlay brine sand layer, where: Vp_1 = 2105 m/s, Vp_2 = 2196 m/s, Vs_1 = 1035 m/s, Vs_2 = 1215 m/s, rho_1 = 2.10 gr/cc and rho_2 = 2.15 gr/cc.

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the new computed curve. This procedure is repeated hundreds times until the computed curve matches with the observed anomaly. This kind of trial and error process can be time consuming and not efficient. An optimization procedure can be incorporated in this process to accelerate the matching process in order to speed up its convergence. In this paper we propose the Levenberg-Marquardt optimization technique and use Shuey's approximation (1985) to define reflection coefficient as a function of angle function:

$$R(\theta) = R_o\left(1+\frac{4}{5}\tan^2\theta\right)-\frac{18}{5}\left(\frac{1-2\sigma}{1-\sigma}\right)R_o.\sin^2\theta+\frac{\Delta\sigma}{1-\sigma}.\sin^2\theta \qquad (1)$$

IV. THE LEVENBERG-MARQUARDT OPTIMIZATION ALGORITHM

In mathematics and computing, the Levenberg–Marquardt algorithm provides a numerical solution to the problem of minimizing a function, generally nonlinear, over a space of parameters of the function. These minimization problems arise especially in least squares curve fitting and nonlinear programming.

The primary application of the Levenberg–Marquardt algorithm is in the least squares curve fitting problem: given a set of m empirical datum pairs of independent and dependent variables, (x_i, y_i), optimize the parameters ^{2} of the model curve $f(x,^2)$ so that the sum of the squares of the deviations

$$S(\beta) = \sum_{i=1}^{m}[y_i - f(x_i,\beta)]^2 \qquad (2)$$

becomes minimum.

Like other numerical minimization algorithms, the Levenberg–Marquardt algorithm is an iterative procedure. To start a minimization, the user has to provide an initial guess for the parameter vector, β. In many cases, an uninformed standard guess like $\beta^T = (1,1,...,1)$ will work fine; in other cases, the algorithm converges only if the initial guess is already somewhat close to the final solution.

In each iteration step, the parameter vector, β, is replaced by a new estimate, $\beta + \delta$. To determine δ, the functions $f(x_i, \beta+\delta)$ are approximated by their linearization

$$f(x_i,\beta+\delta) \approx f(x_i,\beta) + J_i\delta \qquad (3)$$

J_i is the Jacobian matrix given by:

$$J_i = \frac{\partial f(x_i,\beta)}{\partial \beta} \qquad (4)$$

It represents the gradient (row-vector in this case) of f with respect to β. At its minimum, the sum of squares, $S(\beta)$, the gradient of S with respect to δ will be zero. The above first-order approximation of $f(x_i,\beta+\delta)$ gives

$$S(\beta+\delta) \approx \sum_{i=1}^{m}\left(y_i - f(x_i,\beta) - J_i\delta\right)^2 \qquad (5)$$

Or in vector notation,

$$S(\beta+\delta) \approx \|y - f(\beta) - J\delta\|^2 \qquad (6)$$

Taking the derivative with respect to d and setting the result to zero gives:

$$(J^T J)\delta = J^T[y - f(\beta)] \qquad (7)$$

Jacobian matrix is stated earlier whose i^{th} row equals J_i, and where f and y are vectors with i^{th} component $f(x_i,\beta)$ and y_i, respectively. This is a set of linear equations which can be solved for δ. Levenberg's contribution is to replace this equation by a "damped version" of equation (7)

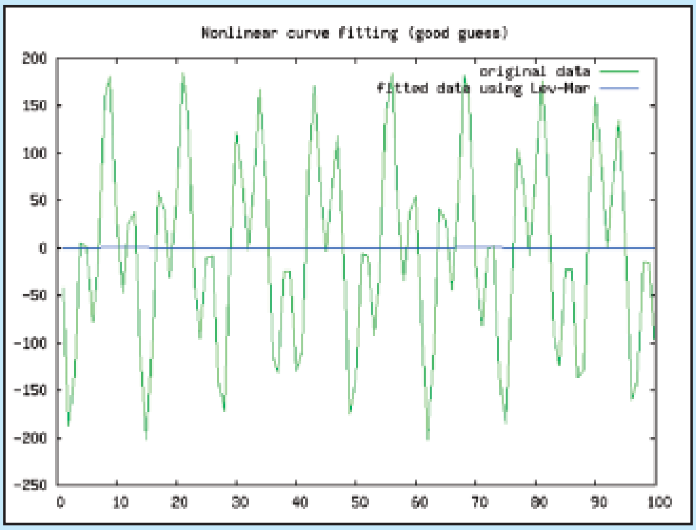

Figure 3
The original function to be fitted

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$$(J^T J + \lambda I)\delta = J^T \left[y - f(\beta)\right] \qquad (8)$$

where I is the identity matrix, giving as the increment, δ, to the estimated parameter vector, β.

The (non-negative) damping factor, λ, is adjusted each iteration. If reduction of S is rapid, a smaller value can be used, bringing the algorithm closer to the Gauss–Newton algorithm, whereas if iteration gives insufficient reduction in the residual, λ can be increased, giving a step closer to the gradient descent direction. Note that the gradient of S with respect to β equals $-2(J^T[y - f(\beta)])^T$. Therefore, for large values of λ, the step will be taken approximately in the direction of the gradient. If either the length of the calculated step, δ, or the reduction of sum of squares from the latest parameter vector, $\beta + \delta$, falls below predefined limits, iteration stops and the last parameter vector, β, is considered to be the solution.

In this example we try to fit the function $y = a\cos(bx) + b\sin(ax)$ using the Levenberg–Marquardt algorithm. The 3 graphs Fig 3, 4, 5 show progressively better fitting for the parameters $a = 100$, $b = 102$ used in the initial curve. Only when the parameters in Fig 3 are chosen closest to the original are the curves fitting exactly. This equation is an example of very sensitive initial conditions for the Levenberg–Marquardt algorithm.

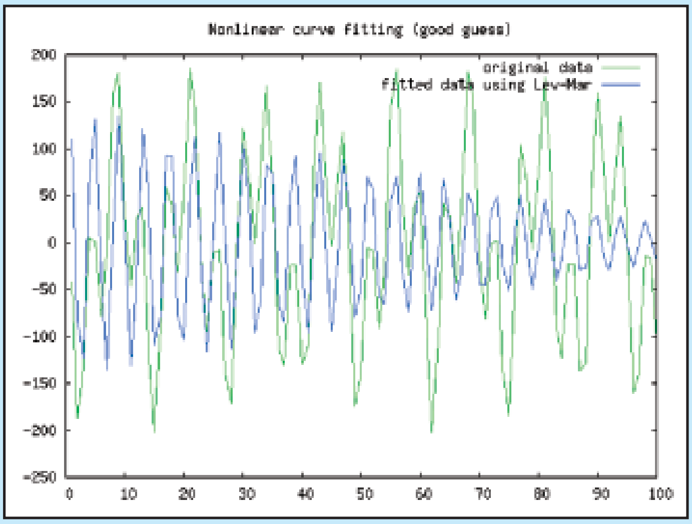

Figure 4
The original function to be fitted \ superimposed by first guess

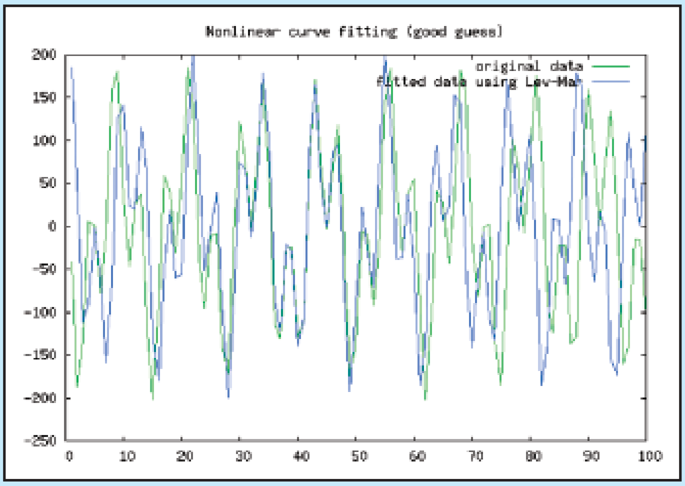

Figure 5
The original function which fits after several iterations and converges quickly using Levenberg-Marquardt optimization technique

VI. RESULT AND DISCUSSION

We provide a standard model data given in Table 1.

Below is flow chart of implementation of AVO inversion using Levenberg-Marquardt optimization technique.

An example of AVO anomaly observed in a CDP gather as an input for AVO inversion is illustrated in Figure 7 (red line). We can see clearly that there is amplitude variation as CDP increases. In other words, there is AVO anomaly when offset increases. It is called anomaly because in normal condition amplitude decreases with offset, but in this data amplitude increases as offset increases.

Figure 8a (1) is an example of amplitude data computed from standard model representing shale over gas sand (expressed in red dot color) and its initial guess of the required parameters (blue dot

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Table 1
Standard model data

Model	Vp (m/s)	Density (gr/cc)	Poisson Ratio
Shale	3000	2.52	0.33
Gas sand	3500	2.00	0.10
Shale	3400	2.55	0.35
Oil sand	3000	2.00	0.20
Shale	3300	2.53	0.34
Water sand	4000	2.20	0.125

color). Figure 8a (2) is an example of perfect matching between data and its fitting using Levenberg-Marquardt optimization algorithm.

Figure 8b (1) is an example of amplitude data computed from standard model representing shale over oil sand (expressed in red dot color) and its initial guess of the required parameters (blue dot color). Figure 8b (2) is the perfect matching using Levenberg-Marquardt optimization algorithm yielding expected petrophysical values.

Figure 8c (1) is an example of amplitude data computed from standard model representing shale over water sand (expressed in red dot color) and its

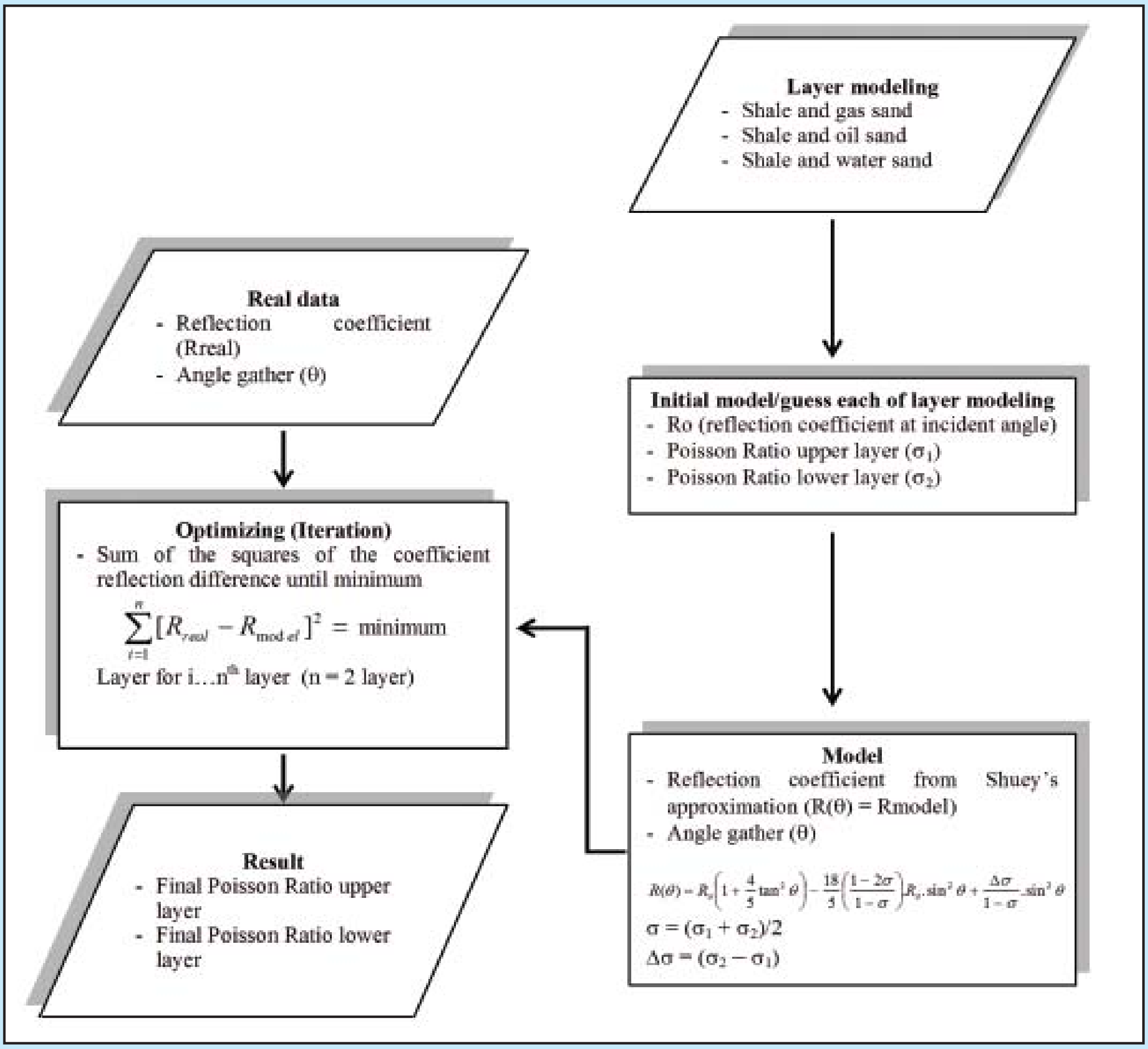

Figure 6
Flow chart of implementation of AVO inversion

initial guess of the required parameters (blue dot color). Figure 8c (2) is an example of perfect matching between data and its final guess using Levenberg-Marquardt optimization algorithm yielding expected petrophysical values.

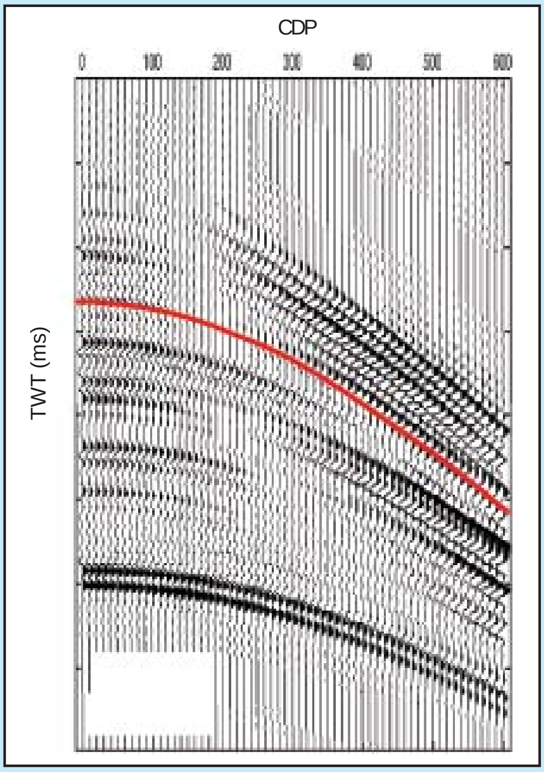

Figure 7
AVO anomaly in a CDP gather

The net result of AVO inversion using Levenberg-Marquardt optimization technique is summarized in Table 2.

It can be seen that AVO inversion using Levenberg-Marquardt optimization technique yields Poisson's ratio which is close to the given model with a relatively negligible error. The Poisson's ratio is the basic element for S-wave prediction which in turn

Table 2
Summary of Levenberg-Marquardt optimization technique

Layer	Poisson Ratio			Error
	TRUE	Guess	Optimi-zation	
Shale	0.33	0.20	0.336	1.7%
Gas sand	0.10	0.30	0.107	6.9%
Shale	0.35	0.20	0.352	0.6%
Oil sand	0.20	0.30	0.205	2.6%
Shale	0.34	0.20	0.342	0.6%
Water sand	0.125	0.30	0.128	2.3%

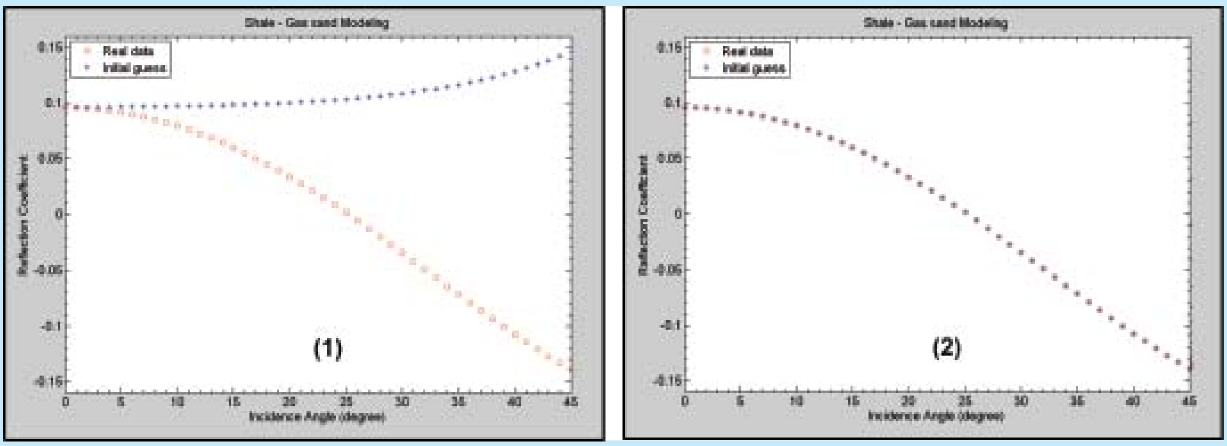

Figure 8a
(1) AVO anomaly (red color) computed form standard model (shale over gas sand) and its first guess (blue dot)
(2) Fitting well after using Levenberg-Marquardtoptimization technique

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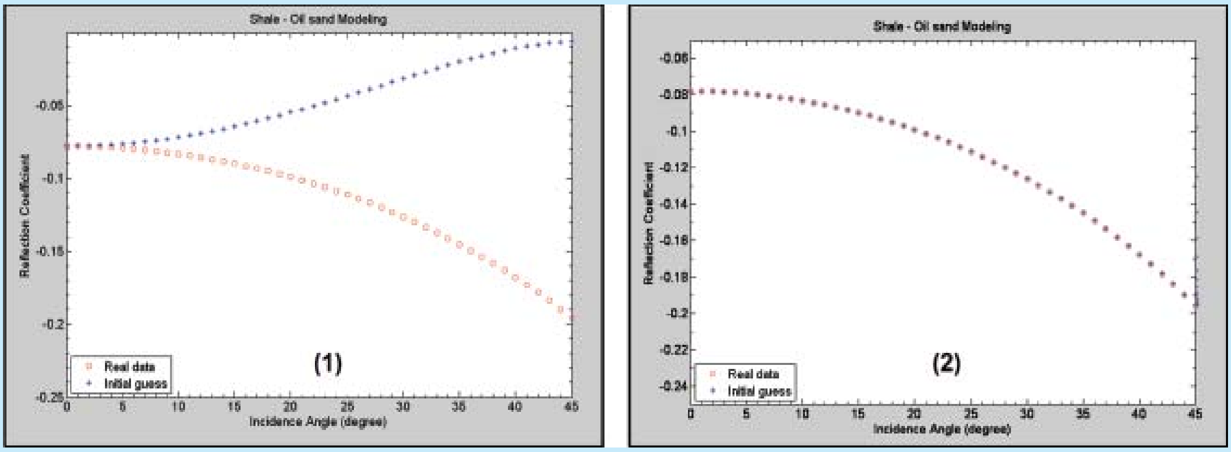

Figure 8b
(1) AVO anomaly (red color) computed form standard model (shale over oil sand) and its first guess (blue dot)
(2) Fitting well after using Levenberg-Marquardt optimization technique

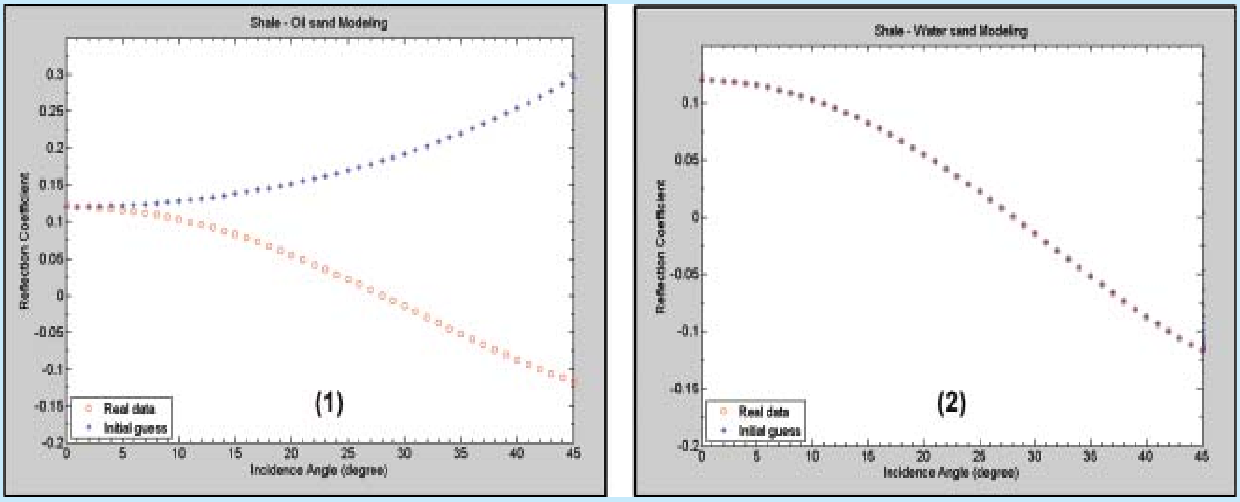

Figure 8c
(1) AVO anomaly (red color) computed form standard model (shale over water sand) and its first guess (blue dot)
(2) Fitting well after using Levenberg-Marquardt optimization technique

can be used for estimating other petrophysical properties.

VII. CONCLUSIONS

1. AVO inversion proves to be an effective tool for estimating petrophysical properties from P-wave seismic data.
2. The inversion process can be tedious if matching AVO anomaly with the model data is not managed wisely.
3. The Levenberg-Marquardt optimization technique has proved to be an effective way for accelerating the matching process because it enables the convergence to be reached very quickly.

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