A NEW TYPE OF DIGITAL BAND – PASS FILTER

by

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ABSTRACT

Digital filtering is mathematical process which provides a means of filtering data numerically. The filtering process can be carried out in the frequency domain to discriminate signal and noise based on their frequency differences.

The digital band-pass frequency filtering has been effectively accomplished using a new type of digital filter. This filter is the result of convolving a boxcar spectral window with a frequency shifted Gaussian function which produces a very smooth transition between the pass-band and the stop-band region. The slope of the filter is controlled by the exponential coefficient of the Gaussian function. For a very narrow pass-band the exponential coefficient also controls the resolution of the filter.

Experiments have been done with the Synthetic Vertical Seismic Profiles data to prove the performance of the filter and shows a good results.

I. INTRODUCTION

Seismic field data are normally recorded through analogue filters to exclude undesired noise. For example, a low cut filter is used to attenuate ground roll and a high cut filter is used to discriminate against wind noise. Often a notch filter is used to reject power line interference. All digital seismic data acquisition systems also incorporate an anti-alias filter to avoid sampling problems (q.v. section II.*B*.). These electronic filters have the effect of improving the signal to noise ratio (S/N ratio) and thereby allow full exploitation of the available dynamic range of the seismic recorder.

A bonus in using the analogue filters mentioned above is that the interpretability of events on the field monitor is enhanced. However, analogue filtering is not sufficient for detailed seismic interpretation. Digital filtering must also be carried out in the processing centre.

Digital filtering offers advantages over analogue filtering in the areas of accuracy, stability and flexibility. It can also be accomplished without phase distortion, an undersirable consequence of all analogue filtering operations.

Digital frequency filtering is entirely a mathematical process. It provides a means of filtering data numerically in either the time domain or the frequency domain. In the time domain, filtering is performed by summing weighted samples at successive time increments (Robinson and Treitel, 1964 ; Sheriff, 1973 . Al Sadi, 1980). This is in effect a convolution between the sampled seismic trace with a filter operator which is called the impulse response of the filter. Since convolution between two time function is equivalent to multiplication of their Z–transforms, the process of digital frequency filtering can be done more efficiently using the Z–transform.

The use of the Z-transform has led to the innovation of a much more efficient filtering technique which is known as recursive filtering (Golden and Kaiser, 1964 ; Shanks, 1967 ; Mooney, 1968). Not all filtering can be done recursively. A considerable effort is required to find the root of high-order and complicated rational polynomial in Z.

Since the advent of the Fast Fourier Transform (FFT) algorithm (Cooley and Tukey, 1965), digital frequency filtering can be carried out in a much more efficient manner. The FFT enables the sampled seismic data to be transformed from the time domain to the frequency domain and vice versa very rapidly. Specification of the filter response as well as the filtering operation can normally be implemented much more conveniently in the frequency domain. The inverse transformation of the filtered spectra back to the time domain is also a straight forward process.

The purpose of this paper is to explain the essence of digital filtering as applied to the processing of VSP data. It is not our intention to give an elaborate discussion on filter theory. An exhaustive literature already exists (see for example : Gold and Rader, 1969 ; Rabiner and Rader, 1972 ; Ackroyd, 1973, Bogner and Constantinides, 1975 ; Oppenheim and Schaffer, 1975 ; Rabiner and Gold, 1975).

We propose a new effective type of band-pass filter which is the result of convolving a boxcar spectral window with a frequency shifted Gaussian function.

The increase of the S/N ratio resulting from digital frequency filtering yields a clear identification of the first breaks and other arrivals in the VSP record. More importantly, it can be used to secure the f-k spectra free from the effect of velocity/spatial aliasing. As an additional benefit with the increase in the S/N ratio, the degree of coherency of the events is also improved, which aids visual correlation of reflections.

II. DIGITAL FILTER SYNTHESIS

A. Overview

Frequency filtering is a process of discriminating signal and noise based on their frequency differences. Each trace in the VSP record contains various frequencies including "noise" which obscures the signal of interest. Therefore, we need to design a specific process which is able to reject certain frequencies.

The traditional approach in designing a digital frequency filter involves finding a set of difference equations having frequency response which significantly resembles a known analogue system function (Rader and Gold, 1967). This is because much information is available on analogue filter design. For example, the Butterworth and Chebyshev analogue filters have found wide application.

Another common approach is through bilinear transformation which uses conformal mapping to transform a digital filter design problem into an analog filter design problem. One attraction of this approach is that it takes into account the fact that the impulse response of an ideal filter has an infinite length (Infinite Impulse Response). Here, the filtering operation is usually performed in the time domain by recursion.

Another popular approach, which is simple to implement, is to truncate the impulse response at a certain length (Finite Impulse Response) but to minimize the undesirable effect of truncation (q.v. section II.C.).

In designing our filter, we will use the Finite Impulse Response approach. In this case, the filtering operation is best performed in the frequency domain and involves the following steps :

1. Design the filter response in the frequency domain.
2. Transform the seismic data into the frequency domain using FFT.
3. Multiply the result of step 1 and step 2.
4. Inverse transform the result of step 3 back to the time domain using inverse FFT.

Care must be exercised in each step, paying particular attention to the repetitive properties of the FFT. Two problems that should be avoided are aliasing and ringing. They are described in the following two sections.

B. Aliasing Consideration

Suppose a continuous seismic trace is represented by a sampled time function consisting of N samples with equal sampling interval T. Let r be an arbitrary integer which denotes time rT.

It is well known that the discrete Fourier transform evaluated at sample number r is equal to the same transform evaluated at sample number r + N (see for example Brigham, 1974). That is, the transform of a sampled function is periodic with a period of N frequency samples.

The length of the sampled seismic trace (time function) can be expressed in the unit of times as

$$T_o = NT \text{ seconds} \tag{1}$$

T_o can be assumed as one period of a periodic waveform, so that the angular frequency can be expressed as

$$\nu = \frac{2\pi}{T_o} = \frac{2\pi}{NT} \text{ rad/sec} \tag{2}$$

Thus, the periodicity of the discrete Fourier transform can be expressed in the unit of frequency as

$$\omega_o = N\nu$$

$$= \frac{2\pi}{T} \text{ cps} \tag{3}$$

Equation (3) shows that the sampling interval establishes a repetitive spectra with a period of ω_o. By nature, the seismic data should have a maximum frequency of ω_m.

Aliasing occurs if

$$\omega_m > \frac{\omega_o}{2} \tag{4}$$

In this situation, spectrum at frequencies greater than $\frac{\omega_o}{2}$ will be folded back or aliased into an interval from 0 to $\frac{\omega_o}{2}$. This ambiquity in representing certain frequencies causes failure of the inverse FFT to properly recover the original signal. In other words, if aliasing occurs, the spectrum is corrupted before the filtering process is applied.

The limit of $\frac{\omega_o}{2}$ is called Nyquist frequency. In designing a digital frequency filter, the highest frequency value is always put well inside the Nyquist frequency.

C. Ringing/Sidelobe Consideration

The sampled seismic trace can be considered as a multiplication of a continuous signal by a "comb" function (see Kanasewich 1981, p. 110), the comb being a series of impulses with equal spacing T. Since we truncate the comb at N samples, the process is equivalent to multiplication of the comb function by a rectangular or "boxcar" function or rectangular window of a width T_o (see equation 1).

The Fourier transform of a boxcar function is a sinc function (see for example Bracewell, 1965; Brigh am, 1974 ; Kanasewich, 1981). Since multiplication in the time domain is equivalent to performing convolution in the frequency domain, the process of convolution between the sinc function and every frequency component of the Fourier transformed seismic data takes place. As a result, a series of spurious peaks (associated with secondary maxima of the sinc function) appears. This spurious peaks are referred to as ringing or sidelobe effect. It arises from a sharp cut-off or sharp-edge in one domain.

The best known way to reduce ringing is to apply a window or gentle taper to the data. Windowing is in effect smoothing the rise and fall of the function around the cut-off values. By windowing, the signal is disturbed slightly, but the spurious oscilation can be suppressed. This technique is therefore a compromise. There is no straightforward procedure to derive the shape of the best window (Bath, 1974 ; p. 155 ; Papoulis, 1977, p. 383). A systematic discussion on several different types of window can be found in a number of textbooks, such as Bath (1974); p. 155, Kanasewich (1981), Rabiner and Gold (1975), Papoulis (1977); p. 383 and Oppenheim and Schafer (1975).

For windowing purposes, we choose the Gaussian function defined by (Champeney, 1973, p. 22).

$$W(t) = e^{-\alpha^2 t^2} \tag{5}$$

where α is a positive constant which controls the slope of the exponential decay. One of the reasons for using a Gaussian function is simply that its Fourier transform is also a Gaussian function which has no tendency to oscilate. The Fourier transform of equation (5) is given by

$$G(\omega) = \frac{\sqrt{\pi}}{\alpha} e^{-(\omega^2/4\alpha^2)} \qquad (6)$$

where w is the angular frequency.

III. THE NEW BAND-PASS FILTER

A. Design Criteria

Our main purpose in digital frequency filtering is to restrict (window) the frequency range of interest. In this case, the windowing process automatically acts as a filter since it rejects a certain frequency band. It is obvious that the improvement of the S/N ratio can only be achieved if the frequency band if the noise falls outside the pass-band region.

For the reason given in section II, the frequency window should be designed as a band-pass filter with the following requirements:

1. To pass a certain frequency band.
2. To suppress ringing and to avoid aliasing.
3. To operate without introducing time delay or phase distortion to the signal.
4. To be a frequency domain operation.

In order to satisfy requirement 1, the frequency range of the desired signals can be multiplied by one (pass-band region) and the frequency range of the unwanted signals can be multiplied by values which are very close to zero (stop-band region). The requirement 2 can be achieved by tapering the transition zone between pass-band and stop-band. To satisfy the requirement 3 and by considering the periodicity of the spectra, the filter response should be real and symmetrical. The requirement 4 can be satisfied by transforming the sampled seismic trace to the frequency domain using the FFT.

B. Filter Slope

An ideal band-pass filter is a rectangular/boxcar spectral window or the Daniel window (Kanasewich, 1981), but due to the properties discussed in subsection II.*B*., it is not wise to simply assess the filter performance in the frequency domain.

We propose a new type of band-pass filter which accommodates the frequency domain and time domain requirements in an optimal sense (see subsection III.*C*.). The principle is obvious and the implementation is quite simple, but, to the best of my knowledge this type of band-pass filter has not been published elsewhere.

Consider the convolution of a boxcar spectral window (Figure 1.a) with a normalized Gaussian spectrum (Figure 1.b). (Note that in Figure 1 (b) the centre frequency has been shifted to 100 Hz in order to display the symmetry of this spectrum). The result of the convolution can be seen in Figure 1 (c). A very smooth transition exists between the pass-band and the stop-band region. The slope or filter taper is controlled by α, the exponential coefficient, which has been set equal to 50 in this example. The smaller α, the steeper the slope of the Gaussian Window. In this filter (bandwidth 60–260 Hz), an α value of 50 corresponds to 20 dB/octave rejection rate in the low-cut region and -90 dB/octave rejection rate in the high-cut region. Each occupies a fractional bandwidth of about 0.2.

Confirmation that this type of band pass filter does not cause strong ringing/sidelobes in the time domain can be shown as follows. Suppose the boxcar spectral window has a band-pass of f_s and its central frequency is f_o. The inverse Fourier transform is given by

$$b(t) = \frac{\sin \pi f_s t}{\pi t} \cos 2\pi f_0 t \qquad (7)$$

The inverse Fourier transform of the normalized Gaussian spectrum is given by.

$$g(t) = \frac{\alpha}{\sqrt{\pi}} e^{-\alpha^2 t^2} \qquad (8)$$

The impulse response of our filter can be obtained by multiplying equations (7) and (8). Equation (7) implies an infinite time extent. But, upon multiplying by the exponential factor in equation (8), the highest energy concentration is only found over a very limited extent. This explains, that if in practice truncation of the impulse response of our filter is unavoidable, the effect has been greatly reduced. This is because we truncate the values which are very close to zero.

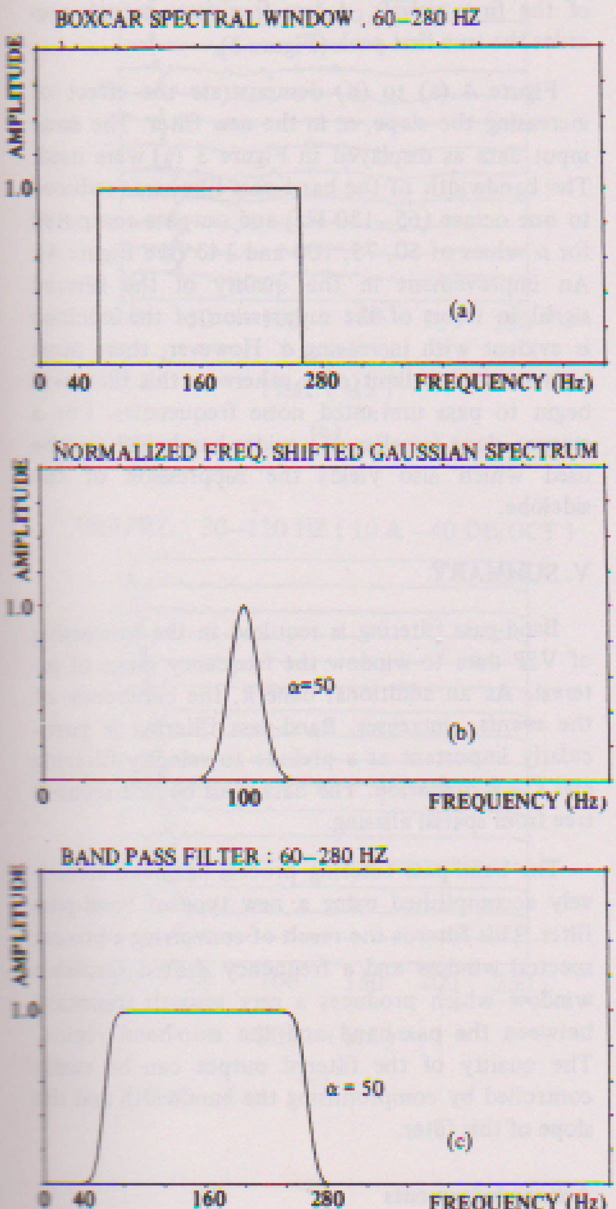

Figure 1

The new type of band-pass filter (c) obtained by convolving the boxcar spectral window (a) and the normalized frequency shifted Gaussian spectrum (b)

C. Resolution

Suppose the low cut-off frequency is f_L and the high cut-off frequency is f_H. The greatest resolution of our band-pass filter is achieved when $f_L = f_H = f_o$. In this case, it becomes a frequency-shifted Gaussian spectral window, a similar window which was used by Dziewonski et al. (1969) in their "multiple filter" group velocity dispersion technique. There, the window was very narrow to separate single frequency arrivals in a dispersed wavetrain. The resolution is controlled by the slope of the exponential curve, α.

Improved resolution in one domain causes the inverse effect in the other domain. The advantage of the Gaussian function is that the product of the RMS duration, $D_\omega.D_t$, in the frequency and time domain remains constant (see Papoulis, 1962, Section 4.4). The frequency-time resolution which can be measured as $1/D_\omega.D_t$, which is greater for the Gaussian function than that for any other type of non-band-limited function. Therefore, the Gaussian function can be considered as equivalent to the optimum filter function (Dziewonski et al., 1969).

For the practical filtering purposes, the resolution is limited by the quality of the filtered signal that can be obtained. Therefore, a compromise should be sought between the bandwidth and the slope of the Gaussian function. For a fixed pass band, the sidelobe effect on the filtered signal can be reduced by decreasing the slope α. A steeper slope can be tolerated on a wide pass-band than on a narrow pass-band filter.

IV. TEST USING SYNTHETIC DATA

The new type of band-pass filter described in section III has been tested using synthetic VSP data. The synthetic data were formed by adding two sinusoidal noise waves to the noise-free synthetic traces, computed using a source signal with a dominant frequency of 100 Hz (Figure 2). The first sinousoidal wave is a low frequency (10 Hz) noise with a signal to noise ratio (relative to Figure 2) equal to 2. The second sinusoidal wave is a high frequency (200 Hz) noise with a S/N ratio equal to 5. The result of the superposition can be seen in Figure 3 (a).

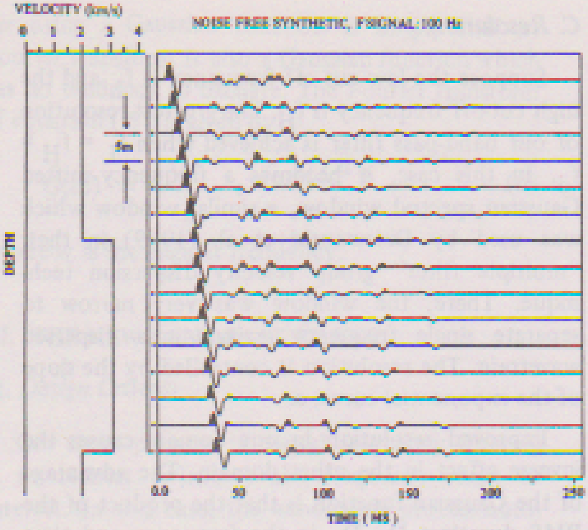

Figure 2

Noise-free synthetic VSP seismograms for a simple three layer velocity model. The dominant frequency of the signal is 100 Hz. Note the presence of both upgoing and downgoing waves. The diagram includes primaries and multiples up to order 2

To demonstrate the effectiveness of the filter, the result has been compared to the boxcar spectral window alone and to the effect of using a trapezoidal spectral window. A trapezoidal filter is specified in terms of its cut-off frequencies (low–cut and high–cut) and cut-off slopes (dB/octave), assumed straight. For comparison purposes with the trapezoidal filter, the same bandwidth and slope were used. For example, for a band-pass of 30–120 Hz and $\alpha = 50$ in our filter, the closest trapezoidal filter has a slope of 10 dB/octave in the low-cut region and –40 dB/octave in the high-cut region.

Figure 3 (a) illustrates the input data used in this evaluation. A hand-pass of 30–120 Hz (two octaves) has been chosen. Figure 3 (b) is the filtered output after applying the boxcar spectral window. Figure 3 (c) is the output after applying the trapezoidal filter. Figure 3 (d) is the output after applying the new filter with $\alpha = 50$.

It can be seen that these three types of filter remove the additive noise. Also, the coherency of events is greatly enhanced. The result of trapezoidal filter is better than that of the boxcar filter. The sidelobe effect evident in Figure 3 (b) has been greatly suppressed in Figure 3 (c). Comparing the trapezoidal filter result (Figure 3 (c) with that of the new filter (Figure 3 – d) we observe improved performance with the latter. The sharp corner in the trapezoidal filter has introduced distortion of the first arrivals. A low frequency trough precedes the true first peak (Figure 2).

Figure 4 (a) to (d) demonstrate the effect of increasing the slope, α, in the new filter. The same input data as displayed in Figure 3 (a) were used. The bandwidth of the band-pass filter was reduced to one octave (65–130 Hz) and outputs computed for α values of 50, 75, 100 and 140 (see Figure 4). An improvement in the quality of the filtered signal in terms of the suppression of the sidelobe is evident with increasing α. However, there must be a maximum limit of α otherwise this filter will begin to pass unwanted noise frequencies. For a steeper slope (smaller α) a wider bandwidth can be used which also yields the suppression of the sidelobe.

V. SUMMARY

Band-pass filtering is required in the processing of VSP data to window the frequency range of interest. As an additional benefit, the coherency of the events increases. Band-pass filtering is particularly important as a prelude to velocity filtering and f – k migration. The data must be first secured free from spatial aliasing.

The band-pass filtering process has been effectively accomplished using a new type of band-pass filter. This filter,is the result of convolving a boxcar spectral window and a frequency shifted Gaussian window which produces a very smooth transition between the pass-band and the stop-band region. The quality of the filtered output can be easily controlled by compromising the bandwidth and the slope of this filter.

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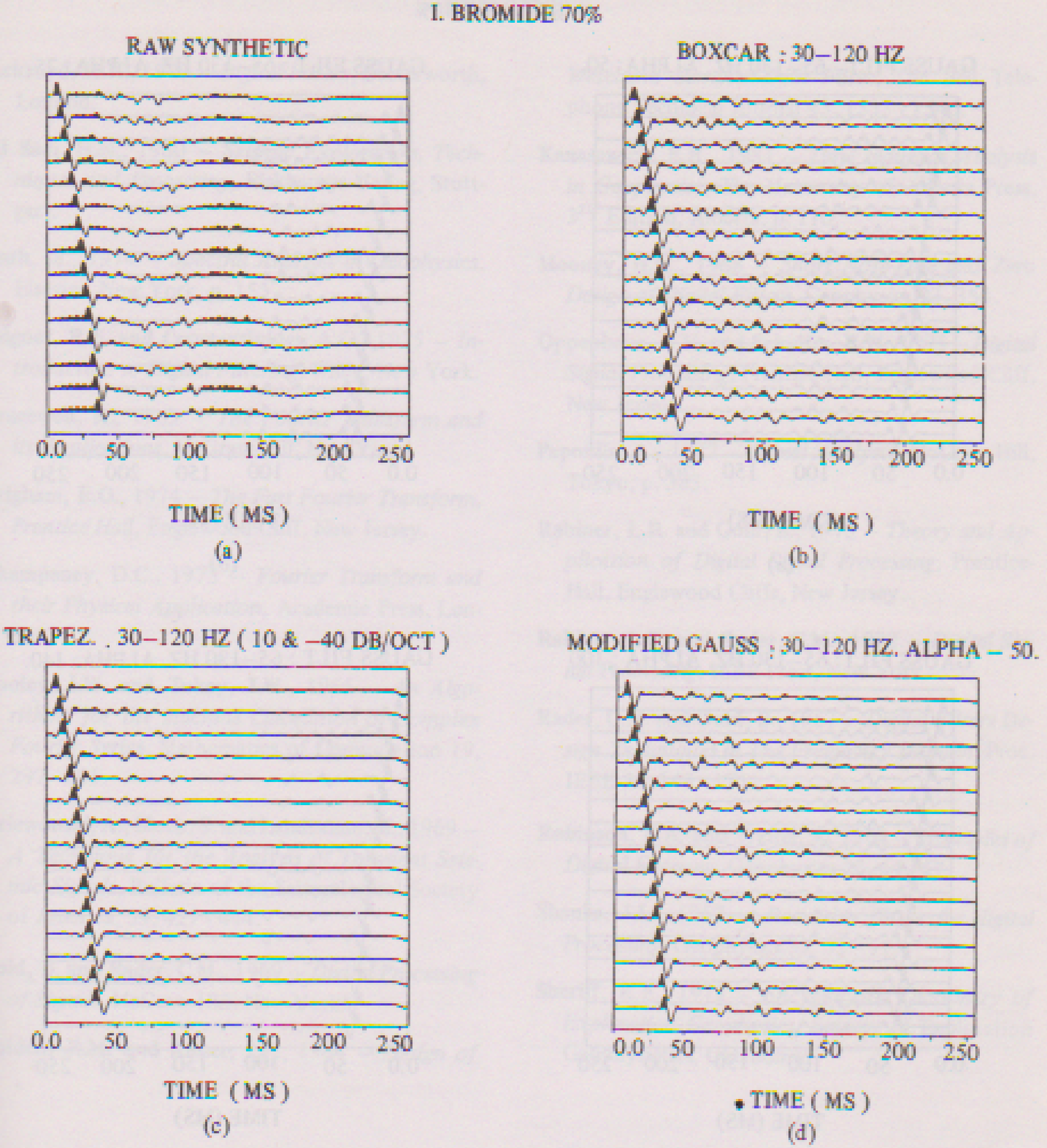

Figure 3

Diagram (a) is the input data. Diagram (b), (c), and (d) are the output after applying the boxcar spectral window, the trapezoidal filter and the new type of band-pass filter respectively

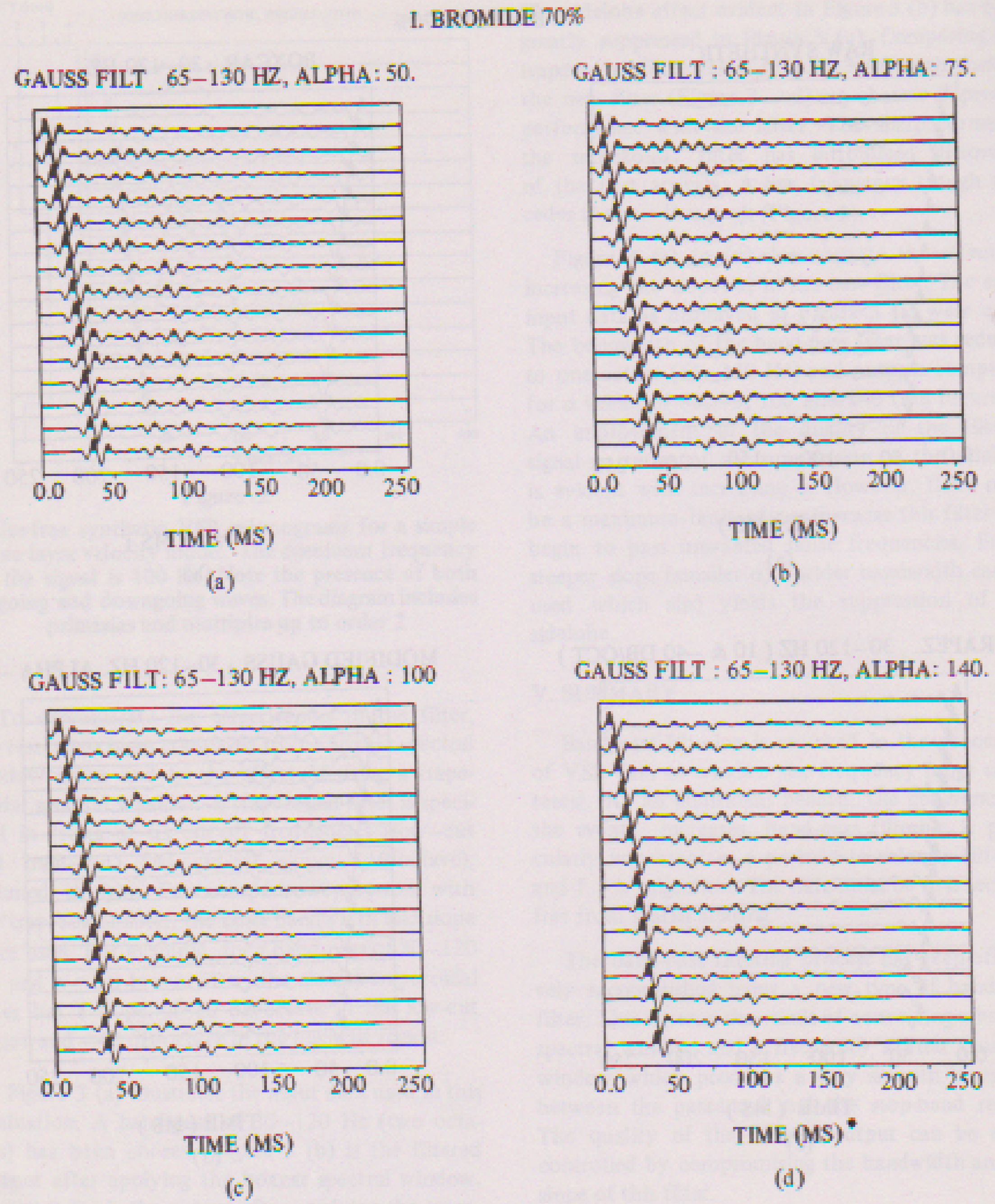

Figure 4

The effect of increasing the slope, α, on the quality of the filter output. Note the decrease of sidelobes. The input data, is shown in Figure 3 (a)

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