FUZZY LOGIC AS A TOOL FOR ESTIMATING PRODUCTION POTENTIAL OF A SAND LAYER

by

Bambang Widarsono, Heru Atmoko, Fakhriyadi Saptono, Tunggal and **Ridwan**

I. INTRODUCTION

In production management, a prior knowledge over production potential of a candidate sand layer (geological complexity in Indonesia has led to existence of stratified reservoirs with a set of layers) to be opened is always desirable. The common practice performed during drilling and completion activities of a production well is through the use of well testing and fluid sampling. From the test, fluid dynamic data such as total liquid rate, water cut, and gas cut are produced. A similar set of data is also required for more mature fields for the purpose of monitoring through the running of routine production and/or swab tests.

Although the tests, especially flow tests during drilling and completion, are always regarded as the only source of proof about productive layer(s) production potential, an alternative means that can be used to provide estimates is always desired. The main reason is that flow tests are costly so that only layer(s) considered as the most potential are to be assigned for testing. Layer(s) that are considered less potential are left untested, eventhough in some cases they are also set on production during the well's production phase.

The idea of establishing a method that can provide illustration over production potentials of all layer(s) always exists. Certainly, there are approaches to serve the purpose such as productivity index (PI) analogy and petrophysical through fractional flow measurement in a core laboratory. However, those approaches are often considered inadequate in accommodating various factors that may influence production potential.

To materialize the idea stated above, the pattern recognition approach was taken. This approach was taken in order to model the relationships between various factors in wellbore and production potential without being trapped by the certain complexity that occurs in any mathematical expressions trying to explain the relationships. For the purpose, fuzzy logic (a branch in Artificial Intelligence) has been used. The choice is actually based on its ability to accommodate both numeric and non-numeric data. Some non-numeric data such as lithology and pore system also have some degrees of influence on production potential.

With a tool that enables us to have production potential estimates of reservoir layers, from which layers with the most promising potential are taken to undergo flow tests. Furthermore, as flow test data has been acquired and used as feedback and calibration by the fuzzy model, production potential of layer(s) with less promising or ambiguous prospect can also be predicted.

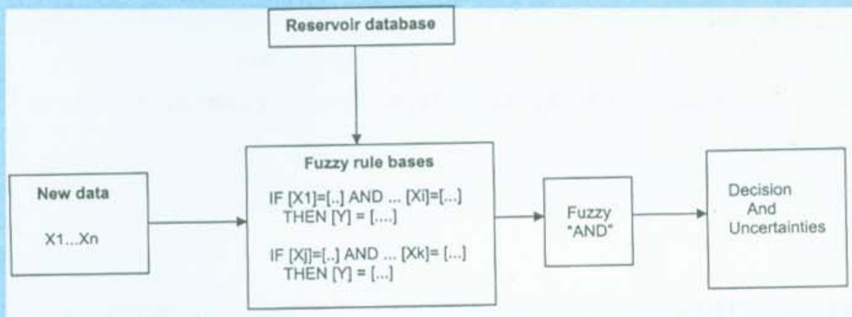
II. METHODOLOGY AND CONCEPT OF FUZZY LOGIC

Unlike the conventional binary or boolean logic which is based on sets of 'true' and 'false', fuzzy logic is essentially a methodology that allows an object to belong to both 'true' and 'false' but with different 'degrees of membership' that range between 0 and 1. This underlines the uniqueness of fuzzy logic that can handle the concept of 'partial truth', which represents truth values that are neither 'completely true' nor 'completely false'.

The most importance of fuzzy logic is that it simulates the way human thinks, i.e. combining quantitative, qualitative, and subjective information. It can handle overlapping, imprecise, approximate, and linguistic information such as 'low', 'medium' and 'high'. This makes fuzzy logic suitable for the nature of data the geophysicists, geologists, and reservoir engineers have to evaluate (Wong et al, 2003).

The relationships between input and output variables can be described in forms of fuzzy IF-THEN rules. Given a set of input-output patterns, many similar fuzzy rules can be derived, in an overlapping manner, to cover the whole functional space. In the process, based on the available "historical" information (i.e. database) that is transformed into fuzzy variables, fuzzy inference rules are constructed and can be easily validated or modified by engineers or geoscientists. When new geological and non-geological factors are obtained, specific decisions can be derived and uncertainties can be quantified in terms of the degree of reliability (i.e. degree of membership). The entire process is best simplified in the schematic diagram presented in Figure 1.

Figure 1
Simplified workflow of the fuzzy logic application



Practically, the 'database' part in the Figure 1 consists of *key data* that is obtained from some *key layers* chosen because of having individual flow test data. (when there is no such data available for a field, data from nearby fields can cautiously be used.) Using this key data, plus additional rock and fluid data for those layers, fuzzy rule bases are extracted from the data population of the *key layers*. One rule for each data pair. Hundreds or even thousands of data pairs mean hundreds or thousands fuzzy rules. The fuzzy rule bases are then reduced in size through eliminating similar fuzzy rules.

The fuzzy interpretation model (i.e fuzzy rule bases) is then ready to be used to estimate production potential (i.e hypothetic individual flow test data) for any *candidate layers* that do not have actual flow test data, either in the same or in other wells. The same type of data variables as the ones used in the modeling are used as input data.

In the process, re-evaluation can be performed when required. The evaluation work in this part may involve re-training of the fuzzy rule bases whenever required or simply addition of new fuzzy rules through external interference from the analyst based on his/her knowledge and experience. Hypothetical data or even data from nearby fields with similarity can also be integrated in the re-training. This also covers evaluation on prediction results for candidate wells in which external modification (i.e. intervention from outside the use of the software package) is a possibility when required.

III. ANALYSIS OF DATA REQUIRED

It is a dream of any production engineer to have accurate production potential data of all layers. Ideally, this data is acquisitioned through the use of direct measurement such as direct flow tests from time to time in a regular basis. This is, of course, often considered as too expensive and impractical, especially for fields with large number of production wells. This is not to mention cases in which the production condition does not suit to the condition required for running the equipment.

Considering the possibility of large number layers and wells, engineers tend to find a means of deriving the needed data through less direct means. Practices, as mentioned before, such as the use of productivity index analogy and hypothetical production flow schemes using fractional flow laboratory test data are often made.

In reality, the examples of practices mentioned above are often considered too simplistic for at least two reasons; Firstly, all two fluids (oil and water or gas and water) or three fluids (oil, water, and gas) contribute to the total well production in exactly the same proportion. This certainly defies some common occurrences in which there are possibilities that decline in production rate are caused by rise in water cut. Secondly, the properties mentioned above are not the only affecting factors on the individual layer's production potential.

In this study attempts have been made to evaluate other factors that may play a significant influence in this matter. Apart from the three rock and fluid properties mentioned previously, rock physical properties such as porosity (fð), water saturation (S_w), irreducible water saturation (S_{wirr}), and shale contents/types. Other factors and data such as geological data (e.g. facies type, clay type, pore type/system, and lithology) may also have some causative pattern with individual layer's production potential. Based on the evaluation, data collection was focused on the data mentioned above. This set of data was then prepared for fuzzy modeling, validation, and prediction.

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IV. REQUIRED KEY DATA

To fulfill the fuzzy modeling, the following data is required from the assigned key layers (in case no data available for a field, data from nearby fields can cautiously be used):

Fluid production data for each individual layer

- Total liquid flow rates
- Oil production rates and water cut

Total production data for key layers

- Total liquid flow rates. The figures must be consistent when compared to the sum of individual layer's flow rates.
- Total oil flow rate and water cut

Supporting data relevant to the estimation of production potential

- Petrophysics: permeability, porosity, water saturation, shale contents, irreducible water saturation.
- Sand geometry: sand thickness, perforation thickness, net sand thickness, sand-shale ratio.
- Geological information: facies, lithology, pore type, shale type
- Engineering data: reservoir pressure, bottom hole pressure (static and dynamic), liquid physical properties.

As a case study a Central Sumatran oil field, CS field, is used. The field was discovered in November 1964 through the exploratory well CS-01, which penetrated an oil column at a depth of 3100 - 4556 ft subsea. Commercial production was started in December 1968. Productive sands of the field are in the Sihapas Group (comprising the Bekasap, Bangko and Menggala formations) and Pematang Group. So far, 92 production wells have been drilled (as per December 2003), of which 79 are in the status of producing.

Table 1
Production (and swab) test data of the key layers

Well	Formation	Reservoir	Data test	Perforated		H (Perf)	Production test			Swab test	
				Top	Bot		BFPD	BOPD	WC (%)	BPH	WC (%)
CS-43	Pematang	50xx SD	14-May-92	5008 5037	5025 5042	22	482	477	1		
CS-44	Bangko	35xx SD	31-May-92	3738 3766	3748 3780	24	254	208	18	27.6	48
CS-54	Pematang	42xx SD	5-Oct-00	4264	4269	5	1008	655	35	25.62	75
CS-73	Menggala	39xx SD	17-Mar-02	4262	4270	8	1296	1102	15	25.62	20
CS-74	Bangko	35xx SD	17-Apr-02	3682	3698	16	1144	1050	5	28.8	20
CS-76	Pematang	42xx SD	28-Apr-02	4570	4582	12	1452	871	40	28.8	20
CS-76	Menggala	38xx SD	24-Aug-03	4072	4096	24	2352	1764	25	37.8	20
CS-78	Pematang	43xx SD	22-May-02	4600	4606	6	1363	1036	24	32.4	20
CS-80	Pematang	42xx SD	15-Jul-02	4614	4624	10	1164	1001	14	43.2	10
CS-81	Pematang	44xx SD	15-Jul-02	4750	4758	8	696	501	28	4.8	10
CS-85	Bangko	36xx SD	22-Mar-03	3836	3842	6	579	490	15	20.16	10
CS-88	Menggala	39xx SD	27-Aug-03	4051	4058	7	1210	12	99	31.2	95
CS-90	Pematang	46xx SD	28-Oct-03	4638	4648	10	432	428	1		

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From the 92 wells that have production flow test data only 32 wells have tests on single layer (all production wells are of commingle completion scheme). Apart from the production tests data there is also swab test data that was taken as a source of comparison only. Combined with other data, only 13 data sets have a complete set of data. Table 1 presents the production test data for the 13 *key layers* that was to be taken as key data.

The petrophysical properties data used in this study is the results of log interpretation conducted by the field's operator. For the 13 data points presented in Table 1, the petrophysical data is presented in Table 2.

For fluid properties, Table 3 presents the supporting data of oil and water formation volume factors (B_o and B_w respectively) and viscosities. Properties of water are taken as constant for all sands and wells.

For pressure data, the static bottom-hole pressure (SBHP) is taken as the reservoir pressure (P_r). A problem had risen when it appeared that there is no flowing bottom-hole pressure (FBHP or P_{wf}) data for the entire field. It is known that the difference between the two pressures ($P_r - P_{wf}$) is considered as a factor that also influence split factor. The solution to the problem is to estimate the needed P_{wf} data based on available static fluid level. Not all wells have the data, however. Table 4 presents the data.

Table 2
Log interpretation and petrophysics data of the key layers

Well	Formation	Reservoir	Interpretasi log				
			K	PHI	VSH	SW	Swirr
CS-43	Pematang	50xx SD	9.2165	0.1007	0.382	0.39003	0.39003
CS-44	Bangko	35xx SD	47.8095	0.1749	0.2434	0.67579	0.3382
CS-54	Pematang	42xx SD	0.792	0.048	0.492	0.589	0.417
CS-73	Menggala	39xx SD	0.1444	0.07535	0.501	0.7571	0.52997
CS-74	Bangko	35xx SD	0.543	0.070	0.599	0.557	0.389
CS-76	Pematang	42xx SD	1.738	0.087	0.704	0.545	0.494
CS-76	Menggala	38xx SD	6.316	0.1745	0.7869	0.5607	0.4486
CS-78	Pematang	43xx SD	3.28389	0.19061	0.12315	0.37059	
CS-80	Pematang	42xx SD	2.369	0.181	0.20775	0.432	0.332
CS-81	Pematang	44xx SD	2.369	0.18	0.20775	0.42372	0.32372
CS-85	Bangko	36xx SD	112.533	0.1724	0.3344	0.5794	0.373
CS-88	Menggala	39xx SD	1078.94	0.1875	0.2455	0.9693	0.166
CS-90	Pematang	46xx SD	13.907	0.0928	0.4125	0.74916	0.28284

Table 3
Fluid properties used for the key layers

Well	Formation	Reservoir	FVF oil RB/STB	FVF wtr RB/STB	Vis-oil cp	Vis-wtr cp
CS-43	Pematang	50xx SD	1.15	1	3	0.26
CS-44	Bangko	35xx SD	1.096	1	3	0.26
CS-54	Pematang	42xx SD	1.15	1	3	0.26
CS-73	Menggala	39xx SD	1.113	1	3	0.26
CS-74	Bangko	35xx SD	1.096	1	3	0.26
CS-76	Pematang	42xx SD	1.15	1	3	0.26
CS-76	Menggala	38xx SD	1.113	1	3	0.26
CS-78	Pematang	43xx SD	1.15	1	3	0.26
CS-80	Pematang	42xx SD	1.15	1	3	0.26
CS-81	Pematang	44xx SD	1.15	1	3	0.26
CS-85	Bangko	36xx SD	1.096	1	3	0.26
CS-88	Menggala	39xx SD	1.113	1	3	0.26
CS-90	Pematang	46xx SD	1.15	1	3	0.26

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For geological data, analysis on data has concluded that there are two main depositional environments; tidal channel/bar and estuary. Data for the 13 data sets are listed in Table 5.

Table 4
Pressure data for key layers

Well	Formation	Reservoir	SBHP(psi)	FBHP(psi)	Delta (psi)
CS-43	Pematang	50xx SD	2636.40	1124.27	1512.14
CS-44	Bangko	35xx SD	791.69	656.54	135.15
CS-54	Pematang	42xx SD	1293.86	1129.48	164.38
CS-73	Menggala	39xx SD	1239.98	1141.06	98.92
CS-74	Bangko	35xx SD	766.88	712.55	54.33
CS-76	Pematang	42xx SD	1373.07	647.35	725.72
CS-76	Menggala	38xx SD	1037.28	643.64	393.64
CS-78	Pematang	43xx SD	1370.56	402.50	968.06
CS-80	Pematang	42xx SD	1376.58	666.39	710.18
CS-81	Pematang	44xx SD	1392.08	1173.16	218.91
CS-85	Bangko	36xx SD	485.89	1121.39	78.61
CS-88	Menggala	39xx SD	1268.85	504.32	764.53
CS-90	Pematang	46xx SD	1374.00	1286.30	87.70

V. DEVELOPMENT OF FUZZY INTERPRETATION MODEL

In the fuzzy modeling, and following the standard steps taken in fuzzy logic, 3 general steps were taken:

1. Data fuzzification
2. Fuzzy inference
3. Defuzzification

Figure 2 presents the process.

Data fuzzification

The first step in preparation of fuzzy modeling was to determine fuzzy sets into which all data variable to be used is grouped. In the process, the *crisp* key data that is available to the study is fuzzified or transformed from numeric to linguistic form. This transforms the Boolean logic that characterizes *crisp* data to fuzzy logic. To underline the difference between the two logics a classification of porosity population serves as an example. For instance, porosity data is grouped into:

LOW porosity < 18%
MEDIUM 18% ≤ porosity ≤ 30%
HIGH porosity > 30%

Unlike the Boolean approach (*crisp* approach) which uses the above classification set in a discontinuous way the fuzzy approach takes the class limits as merely a rough approximation for a continuous data such as porosity. In other words, crisp approach categorizes porosity of 17% as strictly LOW and 18% as strictly MEDIUM whereas fuzzy approach takes both values as belonging to both LOW and MEDIUM but with different degree of membership (see previous section for definition)

Table 5
Depositional environment information for the key layers.
For practical reason Estuary is often omitted in the modeling

Well	Formation	Reservoir	Facies
CS-43	Pematang	50xx SD	Tidal CHN/BAR
CS-44	Bangko	35xx SD	Tidal CHN/BAR
CS-54	Pematang	42xx SD	Tidal CHN/BAR
CS-73	Menggala	39xx SD	Tidal CHN/BAR
CS-74	Bangko	35xx SD	Tidal CHN/BAR
CS-76	Pematang	42xx SD	Tidal CHN/BAR
CS-76	Menggala	38xx SD	Estuary
CS-78	Pematang	43xx SD	Tidal CHN/BAR
CS-80	Pematang	42xx SD	Tidal CHN/BAR
CS-81	Pematang	44xx SD	Tidal CHN/BAR
CS-85	Bangko	36xx SD	Tidal CHN/BAR
CS-88	Menggala	39xx SD	Tidal CHN/BAR
CS-90	Pematang	46xx SD	Tidal CHN/BAR

in regard to each class. Fuzzy sets allow some human expertise and decisions to be modeled more closely.

For the key data discussed above, each data variable was grouped into a set of fuzzy sets with their considered most appropriate membership functions. Figures 3 and 4 present examples of plots of the membership functions for water cut (WC) and shale contents (V_{sh}), respectively. The full membership functions of the two variables are presented in the Appendix. In essence, the functions convert the information from numerical level to linguistic level, as shown in Figure 2. Other numerical variables (porosity, permeability, transmissibility, water saturation, irreducible water saturation, oil rate, and delta pressure) also have their most suitable membership functions, whereas non-numerical variables (lithology, pore type, and depositional environment) are already in linguistic level (for instance, depositional environment: ESTUARINE and TIDAL CHANNEL).

As all data variables had been fuzzified, the next step was to apply fuzzy inference from which the fuzzy determination model would be derived.

Fuzzy inference

Following the approach proposed by Mamdani (Mamdani and Assillian, 1975; Kusumadewi, 2002), after the fuzzification of the *crisp* data, fuzzy rules were generated. These fuzzy rules cover the whole universe of discourse by taking all the possibilities into account. In general, a fuzzy rule takes a form of

$IF\ (x_1\ is\ A_1) \bullet (x_2\ is\ A_2) \bullet (x_3\ is\ A_3) \bullet \ldots\ldots \bullet (x_n\ is\ A_n)\ THEN\ y\ is\ B$

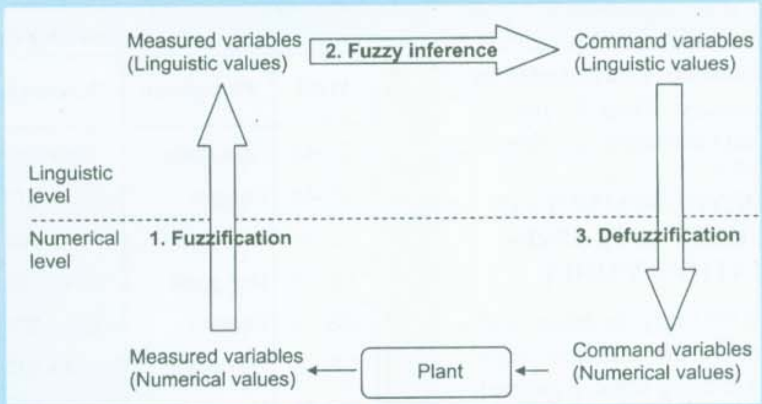

Figure 2
Main components in a fuzzy system (from Wong et al, 2003)

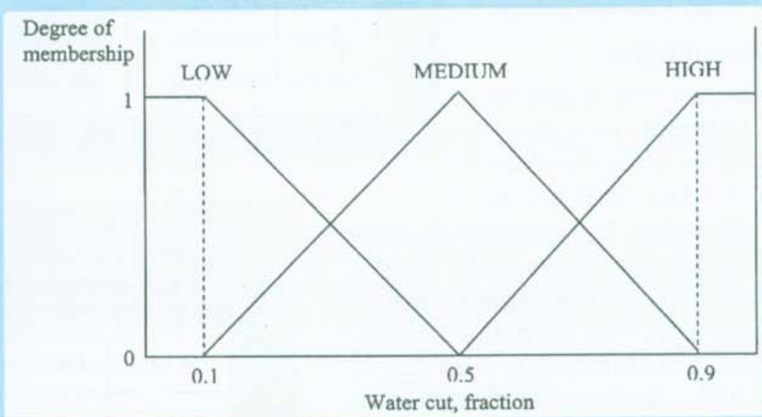

Figure 3
Membership function for water cut

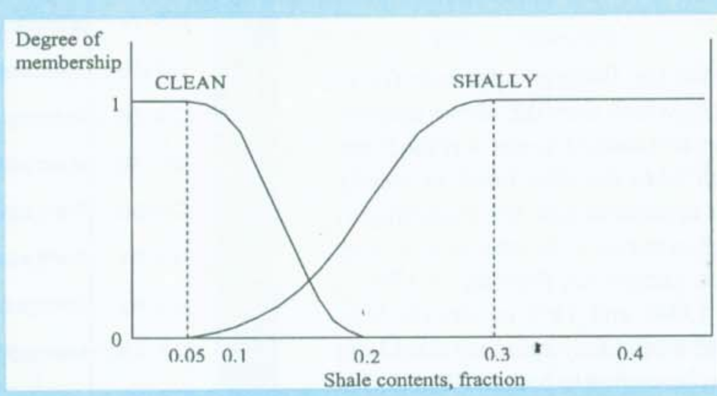

Figure 4
Membership function for shale contents

with propositions following IF are called *antecedent* and propositions following THEN are called *consequence*. In the study the most suitable fuzzy operator is AND, hence making the general rule into

IF (x_1 is A_1) AND (x_2 is A_2) AND (x_3 is A_3) AND …… AND (x_n is A_n) THEN y is B.

In this fuzzy modeling, variables x and y are variables such as porosity, permeability, and water cut whereas A and B are fuzzy statements such as LOW, MEDIUM, SHALLY, etc. An example could be:

IF (K is HIGH) AND (fϕ is HIGH) AND (S_w is LOW) AND …… AND (KH is LARGE) THEN q_o is HIGH

or

IF (K is LOW) AND (V_{sh} is SHALLY) AND (S_{wirr} is MEDIUM) AND …… AND (KH is MEDIUM) THEN q_o is LOW

This rules establishment (fuzzy training) is the center of the fuzzy modeling. A fuzzy model consists of a set of fuzzy rules. For a problem that has two or three relevant variables, the number of fuzzy rules is small. However, for a problem with a large number of variables the number of fuzzy rules is likely to be too large to be established manually. An automatic fuzzy extraction method was used for the purpose.

Further in the inference process, the fuzzy rules undergo operations such as application of *implication* functions and *rules composition* in order to obtain correlation among the rules. Using the established fuzzy rules base, solution will be provided by the model when any sets of data *crisp* are input to it.

Defuzzification

So far, as the fuzzy interpretation model is used, output is still in the form of fuzzy linguistic data (for instance, q_o HIGH and WC LOW). This needs to be transformed back into *crisp* data (for instance, q_o = 615 BOPD and WC = 25%). This is done through a process of defuzzification.

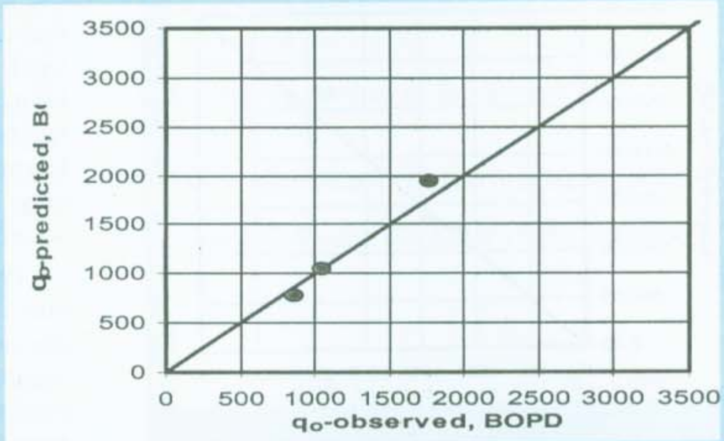

Figure 5
Comparison between observed and fuzzy-predicted individual layer's oil rate

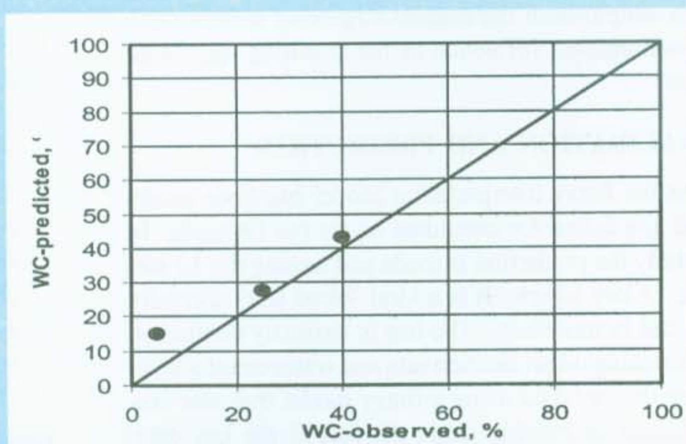

Figure 6
Comparison between observed and fuzzy-predicted individual layer's water cut

There are several known methods proposed to perform the process. The most widely used one is *centroid* method (Kusumadewi, 2002)

$$z = \frac{\int_z z\ \mu(z)\ dz}{\int_z \mu(z)\ dz} \quad \text{or} \quad z = \frac{\sum_{j=1}^{n} z_j\ \mu(z_j)}{\sum_{j=1}^{n} \mu(z_j)}$$

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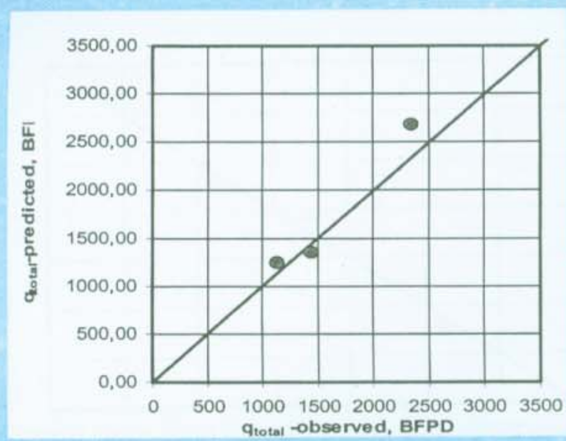

Figure 7
Comparison between observed and fuzzy-predicted individual layer's total fluid rate

with z_j and $\mu(z)$ are the output *crisp* data and its degree of membership, respectively. In essence, the *centroid* method is an aggregation of the various output data, of which, output with the highest degree of membership has predominant influence in the resulting aggregate values.

V. VALIDATION AND PREDICTION

As the fuzzy interpretation model has been established, prediction for candidate layers can be made. In this study the prediction is made just among the 13 sets of the 13 key layers. It is a kind 'blind test' normally practiced in modeling. The test is basically conducted by estimating oil production rate and water cut of a layer among the key data using a fuzzy model that was created based on training using the rest of the key data. For instance, if a test is to estimate oil production rate and water cut of CS -76 (Pematang, 42xx SD) then fuzzy modeling was conducted using the rest of the key data. The estimates were then compared to the real flow test data (observed data).

Three cases were arbitrarily taken, namely wells CS – 76 (Menggala, 42xx SD), CS – 76 (Menggala, 38xx SD), and CS – 74 (Bangko, 35xx SD). Figures 5 through 7 present comparisons between estimated values (oil rate, water cut, and total rate, respectively) and observed values. The results show reasonable agreement between the estimates and the observed values indicating the reliability of the interpretation model.

VI. DISCUSSION

Application of the method in the field will certainly face different problems from one field to another. Problem faced in CS field is that most of the flow tests have been conducted in commingle manner, which causes the number of data sets is limited. In other fields, other problem may occur, such as possession of old log survey data only, absence of reliable fluid and pressure data, and inaccurate completion record.

Problems such as ones mentioned above may reduce the reliability of the fuzzy interpretation model. In this case, experience of the analyst provide a big help to maintain the model's reliability through external inteference on the fuzzy rule bases, a feature that make fuzzy logic an interesting pattern recognition tool. Nevertheless, reliability of any fuzzy models will increase as more actual observed flow test data has become available, so that re-modeling and model modification can be carried out to yield a more robust model.

VI. CONCLUSIONS

From the study, some conclusions have been drawn:

1. Fuzzy logic can be used satisfactorily to estimate production potential of untested reservoir layers.
2. Facility offered by fuzzy logic to enable analysts to incorporate their experience is a great advantage.
3. Data variables used in the fuzzy modeling may be reduced in order to make a simpler model and larger number of fuzzy data training sets. However, this move has a risk of reducing the model's reliability. Balance between data availability and model reliability has to be excercised.

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APPENDIX

The following examples of membership functions are for water cut (triangular model) and shale contents (sigmoid). Other data variables used in the fuzzy modeling may different membership functions, such as trapezoidal and bell-shape, dependent on the variables' data structure and pattern in general. The limiting values in the functions bellow are adopted after some trials. Note the overlapping nature of the functions.

Water cut (WC, fraction)

Membership function: **Triangular** and **Linear**

Data mapping resulted in the division of LOW, MEDIUM, and HIGH.

$$\mu_{LOW}(WC) = \begin{cases} 1; & WC \leq 0.1 \\ (0.5 - WC)/0.4; & 0.1 \leq WC \leq 0.5 \\ 0; & WC \geq 0.5 \end{cases}$$

$$\mu_{MEDIUM}(WC) = \begin{cases} 0; & WC \leq 0.1 \text{ or } WC \geq 0.9 \\ (WC - 0.1)/0.4; & 0.1 \leq WC \leq 0.5 \\ (0.9 - WC)/0.4; & 0.5 \leq WC \leq 0.9 \end{cases}$$

$$\mu_{HIGH}(WC) = \begin{cases} 0; & WC \leq 0.5 \\ (WC - 0.5)/0.4; & 0.5 \leq WC \leq 0.9 \\ 1; & WC \geq 0.9 \end{cases}$$

Shale contents (V_{sh}, fraction)

Membership functions: **Sigmoid** (S-curve), descending and ascending

Data mapping resulted in the division of CLEAN and SHALLY.

For descending S-curve (CLEAN), inflection point = 0.11

For ascending S-curve (SHALLY), inflection point = 0.18

$$\mu_{CLEAN}(V_{sh}) = \begin{cases} 1; & V_{sh} \leq 0.05 \\ 1 - 2[(V_{sh} - 0.05)/0.15]^2; & 0.05 \leq V_{sh} \leq 0.1225 \\ 2[(0.11 - V_{sh})/0.15]^2; & 0.1225 \leq V_{sh} \leq 0.2 \\ 0; & V_{sh} \geq 0.2 \end{cases}$$

$$\mu_{SHALLY}(V_{sh}) = \begin{cases} 0; & V_{sh} \leq 0.05 \\ 2[(V_{sh} - 0.18)/0.12]^2; & 0.05 \leq V_{sh} \leq 0.24 \\ 1 - 2[(0.3 - V_{sh})/0.12]^2; & 0.24 \leq V_{sh} \leq 0.3 \\ 1; & V_{sh} \geq 0.3 \end{cases}$$