

DEVELOPMENT OF A STREAMLINE-BASED HEAT TRANSPORT MODEL FOR THERMAL OIL-RECOVERY SIMULATION

by
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I. INTRODUCTION

Fluid transport calculations based on streamlines have been used successfully for years to model two-phase incompressible flow simulations¹⁾⁻⁴⁾. The pressures for defining the streamlines are obtained by assuming that the reservoir fluids and rock are incompressible and that flow is in the steady state, which yields a time-independent equation that can be solved to define the fixed pressure distribution. Streamline tracking is performed with the pressure field to advance saturations or compositions. In this approach, the changing pressure field and the movement of fluids are not tightly connected, which results in inaccuracies in the solution.

The streamline approach has recently been extended to various applications, such as compositional and black oil problems for updating the composition and saturation⁵⁾⁻⁷⁾. In those cases, a non-linear equation for the pressure is solved assuming unsteady-state flow but compressible fluids and rock, followed by solving the conservation equations in sequence or fully implicitly, i.e. the pressure and the saturation equations are solved together along each streamline. In this approach, most of the physical parameters that depend on the pressure changes are accounted for throughout the solution.

The major limitation of the streamline method is that applicability is restricted to convective problems only. In practice, the contribution of physical diffusion due to gravitational and capillary forces must be considered in modeling a reservoir undergoing a displacement process. The model including diffusion cannot be solved using one-dimensional (1D) streamlines. The operator splitting technique has been proposed to avoid this restriction⁷⁾⁻⁹⁾. The idea is to isolate the convective flow from the diffusion due to gravity for separate solution. The first part is calculated along the common streamline trajectories and the second part is determined by the direction of gravity.

Based on recent advances in streamline-based simulation techniques, we have extended the methods to the thermal oil-recovery simulation. Modeling thermal processes is difficult due to the many complex mechanisms, high degree of non-linearity, and requirements for appropriate thermodynamic formulation to account for the changes in properties with temperature and pressure. The present study approached the problem from a different angle in the streamline framework. An operator splitting technique was applied to handle the heat diffusion due to gravity, capillary, and conduction effects, and the implicit method was used for solving the highly non-linear convective streamline and diffusive equations. A practical rule was introduced to select the time step for pressure updates to reduce the time-lag effects on the coefficients in the phase conservation equations.

A sequential thermal simulator, which solves the pressure and heat equations sequentially, was developed and tested for simulations of hot water-flooding in heavy-oil reservoirs. First we performed simulation with a two-dimensional (2D) heterogeneous reservoir to evaluate the main characteristics of the streamline method such as the number of streamlines, the grid refinement along the streamlines, and the time step size. Then we performed three-dimensional (3D) simulation to examine how the gravity mechanism affects the production performance. The solutions obtained using a commercial thermal simulator¹⁰⁾ were used to compare and validate the developed model.

II. THERMAL OIL-RECOVERY MODEL

Following the previous thermal models^{11),12)}, fluid and heat flow were assumed to obey Darcy's law and Fourier's heat conduction equation, respectively. Additional assumptions are (1) the hydrocarbon component is insoluble in the water phase, (2) the water component is not allowed to dissolve in the oil phase, (3) relative

permeability and capillary pressures are independent of temperature, (4) heat flow consists of convection and conduction only, and (5) heat flow to and from overburden and underburden does not occur. Therefore, the specific forms of the mass conservation equations for the water and oil components are

$$-\nabla \cdot (\rho_w \mathbf{u}_w) + q_w = \frac{\partial(\phi S_w \rho_w)}{\partial t} \quad (1a)$$

$$-\nabla \cdot (\rho_o \mathbf{u}_o) + q_o = \frac{\partial(\phi S_o \rho_o)}{\partial t} \quad (1b)$$

Eqs. (1a) and (1b) have identical form because each phase consists of only one component, and there is no inter-phase transportation of the components. The total energy conservation is given as

$$\begin{aligned} -\nabla \cdot (\rho_w H_w \mathbf{u}_w + \rho_o H_o \mathbf{u}_o) + \nabla \cdot (K_h \nabla T) + q_w H_w^w + q_o H_o^w \\ = \frac{\partial}{\partial t} (\phi S_w \rho_w H_w + S_o \rho_o H_o) + (1-\phi) \rho_r C_r T \end{aligned} \quad (2)$$

The flux velocity, \mathbf{u}_α , is represented by Darcy's law as

$$\mathbf{u}_\alpha = -\frac{kk_{r\alpha}}{\mu_\alpha} (\nabla p_\alpha - \rho_\alpha \nabla D) \quad (3)$$

The mass flow rate, $q_{\alpha k}$, of the α phase from a layer k at wells is given by Peaceman's formulation¹³⁾ as

$$q_\alpha = \frac{2\pi \Delta z_k}{\ln \frac{r_{e,k}}{r_{w,k}} + s_k} \frac{kk_{r\alpha} \rho_\alpha}{\mu_\alpha} (p_k^w - p_{\alpha k}) / (\Delta x \Delta y \Delta z_k) \quad (4)$$

where p_k^w is the pressure in the wellbore, $p_{\alpha k}$ is the phase pressure in the gridblock, $r_{w,k}$ denotes the wellbore radius, $r_{e,k}$ is the equivalent radius, and s_k represents the skin factor.

In addition to the conservation equations, certain ancillary relationships must be specified to calculate the rock and fluid properties. The water-oil and oil-gas capillary pressures are assumed to be functions of phase saturation. Relative permeabilities to water and gas are dependent on the phase saturation. The porosity is taken to be a linear function of pressure as

$$\phi = \phi_{ref} \cdot \exp[c_r (p - p_{sc})] \quad (5)$$

The three-phase oil relative permeability is calculated using the modified Stone's Model II as

$$k_{ro} = k_{rocw} \left[\left(\frac{k_{row}}{k_{rocw}} + k_{rw} \right) \left(\frac{k_{rog}}{k_{rocw}} + k_{rg} \right) - k_{rw} - k_{rg} \right] \quad (6)$$

where $k_{rocw} = k_{row}(S_w = S_{wc}) = k_{rog}(S_g = 0)$.

The phase densities ρ_w and ρ_o are modeled by a correlation function of T and p as¹⁰⁾

$$\rho_\alpha(p, T) = \rho_\alpha^{sc} \cdot \exp \left[c_\alpha (p - p_{sc}) - a(T - T_{sc}) - 0.5b(T^2 - T_{sc}^2) \right] \quad (7)$$

where c_α is the compressibility, and a and b are the thermal expansion coefficients. The phase viscosity may be specified using the following correlation¹⁰⁾

$$\mu_\alpha = a_{vis} \cdot \exp(b_{vis} / T) \quad (8)$$

T is in absolute degrees, whereas a_{vis} and b_{vis} are empirical parameters with values determined from two viscosity measurements at different temperatures. The enthalpies of the water, oil, and gas phases are calculated as follows¹⁰⁾:

$$C_g(T) = c_1 + c_2 T + c_3 T^2 + c_4 T^3 \quad (9a)$$

$$H_{vap}(T) = h_{vap}(T_{cr} - T)^{e_{vap}} \quad (9b)$$

$$H_g(T) = \int_{ref}^T C_g(T) dT \quad (9c)$$

$$H_l(T) = H_g(T) - H_{vap}(T), \quad l = w, o \quad (9d)$$

Here, $C_g(T)$ is the heat capacity of the gas phase expressed by correlation with coefficients c_1 to c_4 , $H_g(T)$ is the gas-phase enthalpy, $H_l(T)$ is the water- and oil-phase enthalpy, and $H_{vap}(T)$ is the vaporization enthalpy expressed by the critical temperature T_{cr} and constants h_{vap} and e_{vap} . Suggested values of those parameters for selected components are given in Reference (10).

III. SEQUENTIAL THERMAL SIMULATOR

The present sequential solution method for thermal oil-recovery simulation differs in several ways from the methods reported in the literature^{14),15)}. The streamline method instead of the finite difference method was applied in the sequential step for solving the energy conservation equations within a time step. Eqs. (1) and (4) are solved simultaneously for pressure and saturation with a Newton finite difference formulation. After these equations are solved, new values for pressure and saturation are obtained at grid point \mathbf{x}_i and the bottom hole pressure of the k -th zone. The obtained pressures are then used to define the streamlines. The algorithm proposed by Pollock¹⁶⁾ is used to lace streamlines through individual cells. The heat convection is then calculated along the streamlines using an implicit single-point upwind scheme to up-

date temperatures followed by diffusion term calculations on reservoir grids. A flow diagram of this simulator is shown in Figure 1.

The main emphasis of this paper is the streamline approach for heat transport, so the solution method of mass transport is not discussed here. Aziz and Settari¹⁷⁾ and Fanchi¹⁸⁾ give more details about the solution methods for fluid transport.

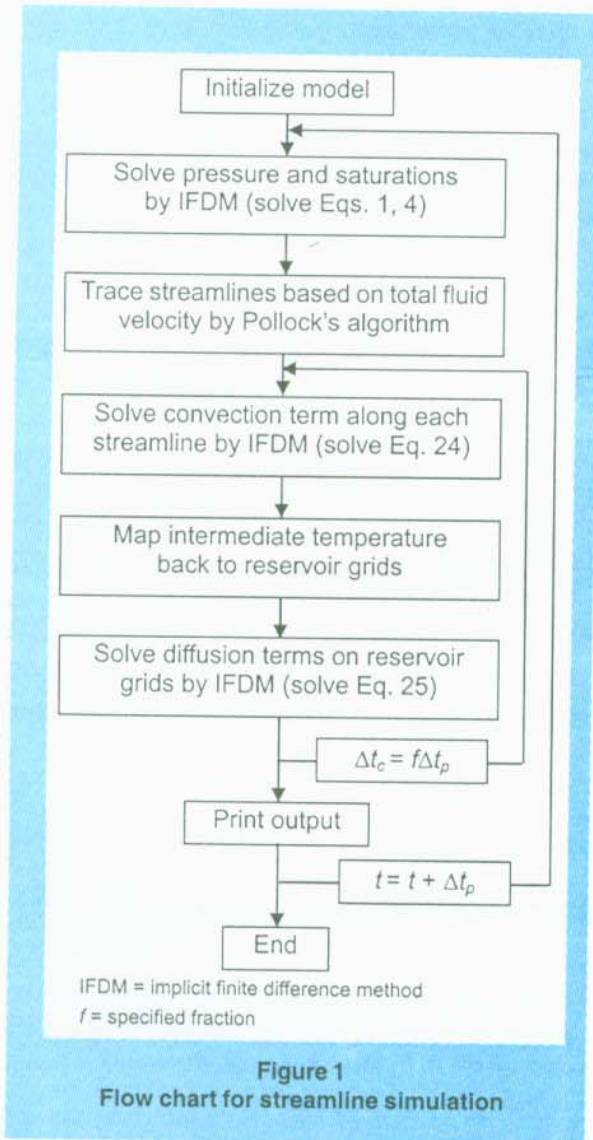


Figure 1

Flow chart for streamline simulation

IV. STREAMLINE APPROACH FOR HEAT TRANSPORT

A hot-water displacement process was formulated to demonstrate the applicability of the streamline method for heat transport. The mathematical model and the solution procedure are illustrated in the following sections.

A. Streamline Energy Equation

To derive the energy equation for streamlines, the total velocity was first defined as a sum of Eq. (3) for the oil and water phases,

$$\mathbf{u}_t = -\frac{kk_{ro}}{\mu_o}(\nabla p_o - \rho_o \nabla D) - \frac{kk_{rw}}{\mu_w}(\nabla p_w - \rho_w \nabla D) \quad (10)$$

The oil- and water-phase pressures are related to the water-oil capillary pressure as

$$p_w = p_o - P_{cwo} \quad (11)$$

Using Eq. (11), Eq. (10) can be rearranged as

$$\mathbf{u}_t = -\frac{k\lambda_w}{\mu_w}(\nabla p_o - \rho_o \nabla D) - \frac{k\lambda_o}{\mu_o}(\nabla p_o - \rho_o \nabla D) \quad (12)$$

where $\lambda_w = k_{rw}/\mu_w$, $\lambda_o = k_{ro}/\mu_o$, and $\lambda_t = \lambda_w + \lambda_o$. Inserting Eq. (3) into Eq. (2), and eliminating p_w and p_o by using Eqs. (11) and (12), the heat transport equation becomes,

$$\frac{\partial}{\partial \tau} \left(S_w \rho_w H_w + S_o \rho_o H_o + \frac{1-\phi}{\phi} \rho_r C_r T \right) + \frac{\mathbf{u}_t}{\phi} \cdot \nabla (f_w \rho_w H_w + f_o \rho_o H_o) + \frac{1}{\phi} \nabla \cdot (k \nabla P_{cwo}) + \frac{1}{\phi} \nabla \cdot \mathbf{G} - \frac{1}{\phi} \nabla \cdot (K_h \nabla T) = \frac{1}{\phi} (q_w H_w + q_o H_o) \quad (13)$$

where

$$f_\alpha = \frac{\lambda_\alpha}{\lambda_t} = \frac{k_{r\alpha} / \mu_\alpha}{k_{rw} / \mu_w + k_{ro} / \mu_o}, \quad \alpha = w, o \quad (14)$$

$$\gamma = \frac{\lambda_w \lambda_o}{\lambda_t} (\rho_w H_w - \rho_o H_o) \quad (15)$$

$$\mathbf{G} = k\gamma(\rho_w - \rho_o) \nabla D \quad (16)$$

Since there is no obvious way to represent the term $\nabla \cdot \mathbf{u}_t$ along the streamline, we assumed $\nabla \cdot \mathbf{u}_t = 0$ when we derived Eq. (13).

The streamline method introduces the coordinate transformation for the convective term in Eq. (13). To do that, we defined the time-of-flight^{16,19)}, τ , along a streamline as

$$\tau(\mathbf{x}) = \int_b^x \frac{\phi}{|\mathbf{u}_t|} d\zeta \quad (17)$$

which leads to rewriting the operator as

$$\mathbf{u}_t \cdot \nabla = \phi \frac{\partial}{\partial \tau} \quad (18)$$

Using the above equation, Eq. (13) can be expressed as

$$\frac{\partial E}{\partial t} + \frac{\partial F}{\partial \tau} + \frac{1}{\phi} \nabla \cdot (k \nabla P_{cwo}) + \frac{1}{\phi} \nabla \cdot \mathbf{G} - \frac{1}{\phi} \nabla \cdot (K_h \nabla T) = \frac{1}{\phi} (q_w H_w^w + q_o H_o^w) \quad (19)$$

Here, E is the total energy per unit volume written as

$$E = S_w \rho_w H_w + S_o \rho_o H_o + \frac{1-\phi}{\phi} \rho_r C_r T \quad (20)$$

and F is the energy convective flow defined as

$$F = f_w \rho_w H_w + f_o \rho_o H_o \quad (21)$$

The 3D convective problem is now transformed into the 1D problem to solve the energy equation, Eq. (19), along the streamlines.

B. Streamline Operator Splitting

Eq. (19) is recognized as a system of convection-diffusion equations with the diffusion terms consisting of capillary, gravity, and conduction effects. By using the technique of operator splitting, we split the diffusion terms on a non-orthogonal grid, locally spanned by the streamlines and by the direction of diffusions. The advantage of decoupling Eq. (19) in this way is that the diffusions can be solved independently and only solved in regions where they are important. Variations on this operator splitting approach have been taken by several authors²⁰⁻²².

Next we describe how to perform temperature calculations with the system given by Eq. (19) using an operator splitting method. Three steps are taken for calculating temperatures at the $n+1$ time step from the temperatures at the n time step.

Step 1: Let $T(\tau, t) = S_\zeta(t) T^n$ be the streamline solution to the convection part of Eq. (19) that is written as

$$\frac{\partial E}{\partial t} + \frac{\partial F}{\partial \tau} = \frac{1}{\phi} (q_w H_w^w + q_o H_o^w); \quad (22)$$

$$T(\tau, 0) = T^n(\tau); \quad T(0, t) = T_{inj}, \quad t > 0$$

If $S_\zeta(t)$ denotes the operator that maps T^n to $T(\tau, t)$, we can define $T^{n+1/2} = S_\zeta(\Delta t) T^n$ as the intermediate solution, where subscript ζ stands for the grid spacing of the streamline solution.

Step 2: Let $T(\mathbf{x}, t) = R_\Delta(t) T^{n+1/2}$ be the finite difference solution to the diffusion parts of Eq. (19) that is written as

$$\frac{\partial E}{\partial t} + \frac{1}{\phi} \nabla \cdot (k \nabla P_{cwo}) + \frac{1}{\phi} \nabla \cdot \mathbf{G} - \frac{1}{\phi} \nabla \cdot (K_h \nabla T) = 0 \quad (23)$$

$$T(\mathbf{x}, 0) = T^{n+1/2}(\mathbf{x}); \quad \nabla T(\mathbf{x}, t) \cdot \mathbf{n} = 0$$

Eq. (23) is solved using N_d time steps of length k_d , i.e., $k_d N_d = \Delta t$. The splitting solution at time t^{n+1} is given by $T^{n+1} = R_\Delta(\Delta t) T^{n+1/2}$, where the subscript Δ stands for the grid spacing of the finite difference solution.

Step 3: Then for a time $N\Delta t$, the approximated solution of Eq. (19) can be calculated with N -times repeated use of the operators as

$$T^{n+1}(\mathbf{x}) = [R_\Delta(\Delta t) S_\zeta(\Delta t)] T^n(\mathbf{x}) \quad (24)$$

C. Numerical Solutions

We employed an implicit single-point upwind scheme along each streamline for Eq. (22) and implicit finite difference scheme for Eq. (23). Applying the implicit single-point upwind scheme for the convective streamline step has some striking advantages. First, the scheme requires no stability conditions, known as CFL (Courant-Fredrick-Lewy) conditions, associated with this method in the sense that time step, Δt , is not restricted by the spatial t discretization parameter. Second, this scheme allows Eq. (22) to be solved on the irregularly spaced t grid instead of the regular grid as when the equation is solved explicitly. On the other hand, the finite difference scheme to solve Eq. (23) slows the calculation because the capillary force and conduction must be solved in three dimensions, although the gravity equation can be solved over the gravity lines.

The implicit scheme for Eq. (22) requires calculations of the accumulation, E , the flux, F , and the fluid enthalpies, H_w^w and H_o^w , for each grid i at $k+1$ time level as

$$\frac{1}{\Delta t_c} [E(T_i^{k+1}) - E(T_i^k)] + \frac{1}{\Delta \tau} [F(T_i^{k+1}) - F(T_{i-1}^{k+1})] - \frac{1}{\phi_i} [q_{wi}^{n+1} H_w^w(T_i^{k+1}) + q_{oi}^{n+1} H_o^w(T_i^{k+1})] = 0 \quad (25)$$

This is a non-linear equation for T_i^{k+1} which can be solved using Newton-Raphson's procedure^{17,18}. Eq. (25) may be rewritten in terms of residual, R_c , which is equal to the left side of Eq. (25), as

$$R_c(T_i^{k+1}) = 0 \quad (26)$$

The solution of Eq. (26) is obtained iteratively. For the first iteration ($v=1$), we use $T_i^v = T_i^k$, where T_i^k is known. Subsequent estimates for the updated temperatures are found as follows. First, we define the difference in temperature between two Newton iterations $v+1$ and v as

$$\delta T_i = T_i^{v+1} - T_i^v \quad (27)$$

δT_i at τ grid point i can be obtained by performing a Taylor series expansion of Eq. (26) with the first order in the small quantity δT ,

$$R_{ci}^{v+1} = R_{ci}^v + \left(\frac{\partial R_{ci}}{\partial T_j} \right)^v \delta T_j \quad (28)$$

The value of δT_j that satisfies Eq. (28), $R_{ci}^{v+1} = 0$, obeys the expression

$$\left(\frac{\partial R_{ci}}{\partial T_j} \right)^v \delta T_j = -R_{ci}^v \quad (29)$$

$\partial R_{ci} / \partial T_j$ is an element of the Jacobian matrix that is obtained numerically or analytically. Substituting Eq. (25) into Eq. (29) yields

$$\begin{aligned} \frac{1}{\Delta t_c} \left(\frac{\partial E}{\partial T} \right)_i^v \delta T_i + \frac{1}{\Delta \tau} \left[\left(\frac{\partial F}{\partial T} \right)_i^v \delta T_i - \left(\frac{\partial F}{\partial T} \right)_{i-1}^v \delta T_{i-1} \right] \\ - \frac{1}{\phi_i} \left[q_{wi}^{n+1} \left(\frac{\partial H_w^*}{\partial T} \right)_i^v + q_{oi}^{n+1} \left(\frac{\partial H_o^*}{\partial T} \right)_i^v \right] \delta T_i = -R_{ci}^v \quad (30) \end{aligned}$$

The solution is considered to converge when these increments of δT are smaller than the convergence criterion. The updated temperatures resulting from the solution of Eq. (30) have to be mapped back to the underlying grid in order to provide the initial condition, $T^{n+1}(\mathbf{x}) = T^{n+1}(\mathbf{x})$, for solving Eq. (23).

Following the procedure to derive Eq. (30), the discretized form of Eq. (23) is obtained in terms of the residual as

$$\begin{aligned} \frac{V_b}{\Delta x_d} \left(\frac{\partial E}{\partial T} \right)_i^v \delta T_i \\ + \Delta_x \left[\frac{k}{\phi} \left(\frac{\partial \gamma}{\partial T} \right)_i^v \Delta_x P_{cwo}^{n+1} \delta T_i \right] + \Delta_y \left[\frac{k}{\phi} \left(\frac{\partial \gamma}{\partial T} \right)_i^v \Delta_y P_{cwo}^{n+1} \delta T_i \right] + \Delta_z \left[\frac{k}{\phi} \left(\frac{\partial \gamma}{\partial T} \right)_i^v \Delta_z P_{cwo}^{n+1} \delta T_i \right] \\ + \Delta_x \left[\frac{1}{\phi} \left(\frac{\partial G}{\partial T} \right)_i^v \delta T_i \right] - \Delta_x \left(\frac{K_h}{\phi} \Delta_x \delta T_i \right) - \Delta_y \left(\frac{K_h}{\phi} \Delta_y \delta T_i \right) - \Delta_z \left(\frac{K_h}{\phi} \Delta_z \delta T_i \right) = -R_{di}^v \quad (31) \end{aligned}$$

The solution of Eq. (31) gives the approximated solution of the system described by Eq. (19) at the $n+1$ time step for node i . The capillary term for x -direction is discretized as

$$\begin{aligned} \Delta_x \left[\frac{k}{\phi} \left(\frac{\partial \gamma}{\partial T} \right)_i^v \Delta_x P_{cwo}^{n+1} \delta T_i \right] \\ = TX_{P_{cwo}^{n+1}/2} (P_{cwo} \delta T)_{i-1} - (TX_{P_{cwo}^{n+1}/2} + TX_{K_{h+1/2}}) (P_{cwo} \delta T)_i + TX_{P_{cwo}^{n+1}/2} (P_{cwo} \delta T)_{i+1} \quad (32) \end{aligned}$$

Similarly, the discrete version of the conduction term for x -direction is

$$\Delta_x \left(\frac{K_h}{\phi} \Delta_x \delta T_i \right) = TX_{K_{h-1/2}} \delta T_{i-1} - (TX_{K_{h-1/2}} + TX_{K_{h+1/2}}) \delta T_i + TX_{K_{h+1/2}} \delta T_{i+1} \quad (33)$$

The coefficients in Eqs. (32) and (33) are given as follows:

$$TX_{P_{cwo}^{n+1}/2} = \left(\frac{k}{\phi} \frac{\partial \gamma}{\partial T} \right)_{i \pm 1/2} \frac{A_x}{\Delta x_{i \pm 1/2}}; \quad TX_{K_{h \pm 1/2}} = \left(\frac{K_h}{\phi} \right)_{i \pm 1/2} \frac{A_x}{\Delta x_{i \pm 1/2}}$$

A difference approach is applied to the gravity term as

$$\Delta_z \left[\frac{1}{\phi} \left(\frac{\partial G}{\partial T} \right)_i \delta T_i \right] = A_z \left[\left(\frac{1}{\phi} \frac{\partial G}{\partial T} \right)_{i+1/2} - \left(\frac{1}{\phi} \frac{\partial G}{\partial T} \right)_{i-1/2} \right] \delta T_i \quad (35)$$

The upstream direction, at which G is to be evaluated, is dependent on the fluids, and is based on the flow direction⁹⁾. For the case of a two-phase water-oil system in which the oil is less dense than water, the proper approximation of $(\partial G / \partial T)_{i+1/2}$ is based on the water properties at node i and the oil properties at node $i+1$. A similar approach is employed for $(\partial G / \partial T)_{i-1/2}$.

In actual applications, the iterative calculations for Eqs. (30) and (31) tend to converge quickly. Our numerical experiments on hot water-flooding showed that the first approximation, T^{n+1} , is sufficiently accurate and that additional approximations are not warranted. The time step for solving Eq. (19) depends on the numerical scheme and the physical parameters considered in the model. This issue remains for further studies, though typical values were found to be 0.2 to 1.0 times the pressure time step in the numerical experiments.

V. MODEL PERFORMANCE

Simulations of hot water-flooding in heavy-oil reservoirs were conducted to examine the performance of the developed simulator. The simulation results were compared with the results of an existing commercial thermal simulator (CMG's STARS) to verify and evaluate the proposed methods.

A. Problem Descriptions

The illustrative problem is a heavy oil reservoir of 14 °API oil that underwent conventional (cold) water-flooding at the reservoir temperature. The modeled region of the reservoir is a quarter of the repeated five-spot pattern with an area of 75'75 m² and thickness of 5 m. The model includes two wells of 0.09 m wellbore radius, an injection well placed at the lower left edge of the region and a pro-

duction well placed at the upper right edge. Fluid and rock properties, and the reservoir and surface conditions are listed in Table 1. The relative permeability curves are shown in Figure 2. Hot water of 150 °C is injected at the constant rate of 100 m³/day cold water equivalent. The producer well is operated under the minimum bottom-hole pressure constraint of 100 kPa. We ran the simulator for 300 days or the equivalent of 3.18 pore volume injected (PVI) with the constraints of the maximum changes in the basic variables allowed over a time step as $(dp)_{\max} = 343.2$ kPa, $(dS_w)_{\max} = 0.2$, and $(dT)_{\max} = 7.5$ °C, to monitor pressure updates as well as to control convergence for solving Eq. (1). This run is named Base-Case.

B. Examples of 2D Problem

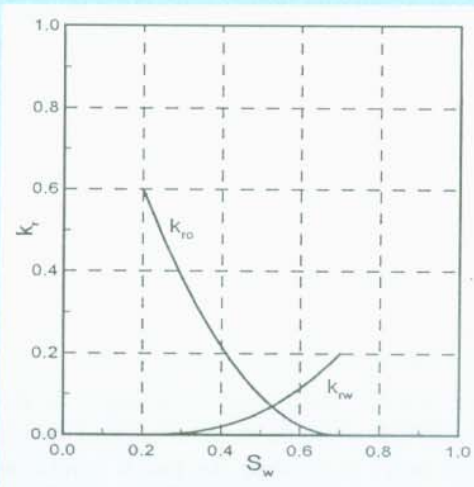
First we performed a simulation of hot water-flooding in a 2D heterogeneous reservoir to evaluate the ef-

fects of the number of streamlines, the grid refinement on streamlines, and the time step size. A grid system of 15x15x1 was employed. Figure 3 shows the permeability field. The pressure and water saturation distributions shown in Figures 4(a) and (b), respectively, were attained by cold water-flooding, and were used as the starting conditions for hot water-flooding. The temperature distribution is nearly uniform at 52 °C.

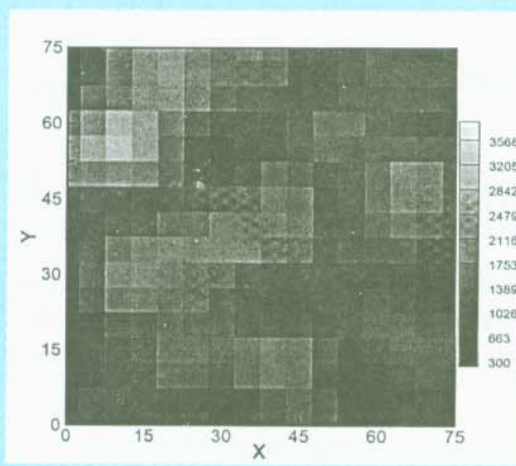
Comparisons of oil production rate, water cut, and oil recovery calculated from this thermal simulator by

**Table 1
Data for Simulation**

Parameters	Water	Oil
Critical properties:		
M , kg/gmol	0.018	0.600
T_{cr} , °C	374.1	493.8
p_{cr} , kPa	2.206E+4	1.118E+3
Density calculation:		
ρ^{sc} , kg/m ³	998	972
c , kPa ⁻¹	4.57E-7	7.30E-7
a , °C ⁻¹	-1.91E-3	3.80E-4
b , °C ⁻²	0.00E+0	0.00E+0
Viscosity calculation:		
a_{vis} , kPa.day	4.361E-8	5.182E-10
b_{vis} , °K	1610.7	5088.9
Enthalpy calculation:		
c_1	32.24	-22.38
c_2	1.924E-3	1.939E+0
c_3	1.055E-5	-1.117E-3
c_4	-3.596E-9	2.528E-7
h_{vap}	25.1	287.1
θ_{vap}	0.380	0.625
Rock thermal conductivity, kJ/(m.sec.°C)		3.5E-3
Reservoir heat capacity, kJ/(m ³ .°C)		2347
Porosity at reference pressure		0.3
Reference pressure, kPa		6890
Reference temperature, °C		15.6
Surface pressure, kPa		100
Surface temperature, °C		15.6



**Figure 2
Relative Permeability Curves**



**Figure 3
Permeability field (mD)
used for 2D Simulation**

streamline (TESIS) and STARS are depicted in Figure 5. The STARS solutions of oil rate and water cut are shown as smooth curves, whereas the TESIS solutions are shown as ragged curves, especially after 200 days. The agreements between both numerical methods are considered satisfactory, especially before 200 days. The departure appears after 200 days when breakthrough of the hot water of about 100 °C has occurred. The lower oil rates of TESIS were most likely a result of the significant contribution of viscosity forces. As illustrated in Figure 6 for the viscosity and density of heavy oil used in this study, the viscosity decreases dramatically with a rise in temperature but the density changes are relatively less. In the sequential method, the new streamlines were obtained using the temperatures at the start of time step, which made the streamline velocity lower. Consequently, the new rate of heat flow was insufficient to attain the high temperature breakthrough. This is a known problem associated with the streamline methods based on an IMPES sequential approach, which causes unstable behavior when the fluid properties are a strong function of pressure.

The sensitivity of the solution to the number of streamlines is also demonstrated in Figure 5. Figure 5(b) indicates that the solution improve with increasing number of streamlines. All these streamline solutions were obtained with the irregularly spaced t grid on the streamlines. The dashed line with square symbol (150 SL-RG) illustrates a solution with the regular t grid using 150 streamlines. The number of nodes for the regularly spaced t was determined as the number of the grid-blocks that a streamline passed through multiplied by two⁹. Transform-

ing onto a regular t grid tends to reduce the computing time of the simulation as confirmed by examples, but becomes a source of numerical diffusion in the streamline method⁹ which would explain the obtained lower oil recovery among others in Figure 5(b). Analysis of the streamline number and the t grid size was primarily done to confirm whether the simulation speed of this model could be improved.

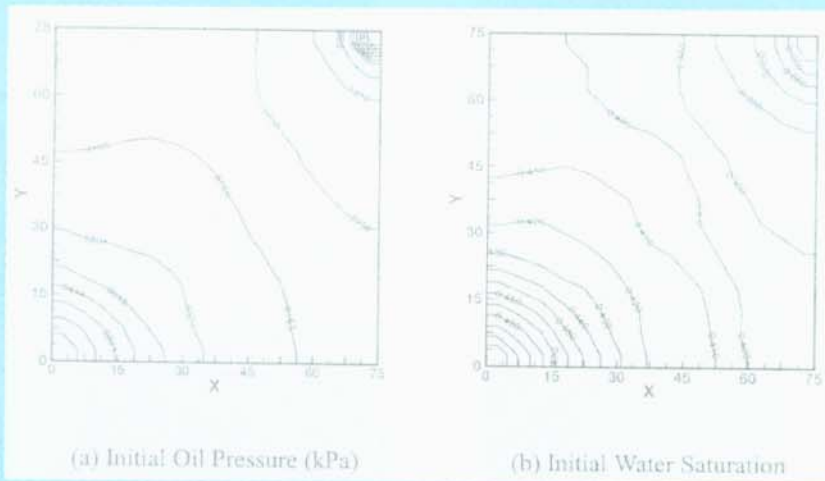


Figure 4
Initial conditions of pressure and water saturation for 2D Case,
(a) Initial oil pressure (kPa), (b) Initial water saturation

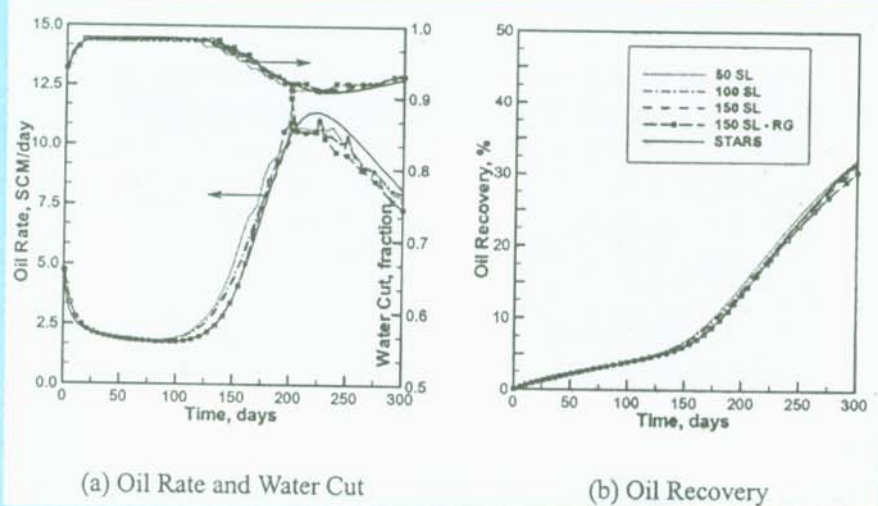


Figure 5
Comparison of production performances of 2D simulation,
(a) Oil rate and water cut, (b) Oil recovery

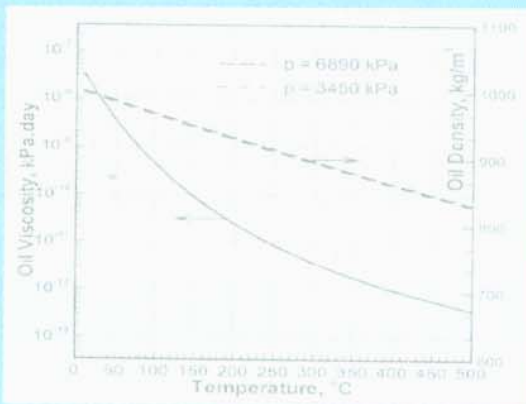


Figure 6
Viscosity and density of heavy oil used for simulation

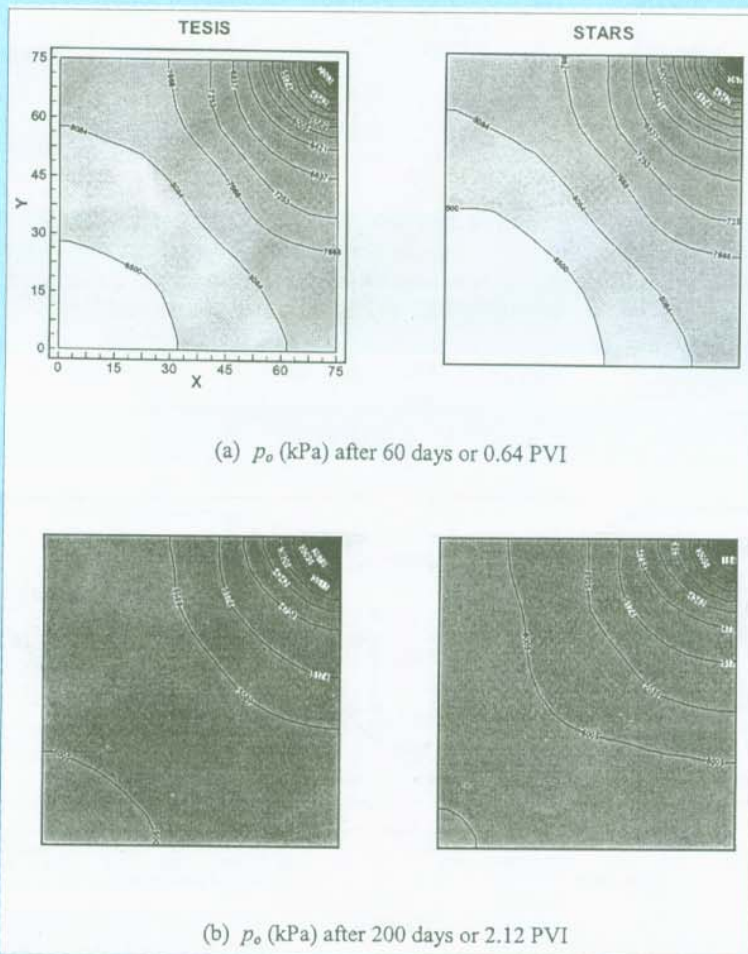


Figure 7
Comparison of pressure distributions of 2D Simulation, (a) p_o (kPa) after 60 days or 0.64 PVI, (b) p_o (kPa) after 200 days or 2.12 PVI

Figures 7-9 present visual comparisons of the pressure, water saturation, and temperature distributions obtained by TESIS and STARS. These figures provide insight into the reasons for the similarities and differences depicted by the production performance curves in Figure 5. The distortion of the fronts of water saturation and temperature was caused by the high heterogeneity of permeability. The presence of a more permeable region at the left top caused the growth of the front zone to deviate from predominant flow direction and veer to the left. This pattern appears to be similar in the results of both models.

Effects of the sequential time step for solving the mass balance and the energy balance equations are presented in Figure 10. In this case, called Case-01, we ran TESIS and STARS using the numerical control (the maximum changes in the basic variables

allowed over a time step) as $(dp)_{max} = 490.3$ kPa, $(dS_w)_{max} = 0.2$, and $(dT)_{max} = 25$ °C. As these values except $(dS_w)_{max}$ were much larger than those for Base-Case, the time step size tended to be larger in Case-01 than in Base-Case. With a larger time step, the lower temperature of the previous time step is used for calculating mobility, and this causes still lower streamline velocity. As a result, the onset of incremental oil production is delayed, and the oil rate curve is shifted to the right leading to a larger difference between TESIS and STARS. In the STARS solution, pressure and temperature are solved implicitly, and completely aligned for each time step. The oil recovery for TESIS was also lower than for STARS. As expected, however, the total sequential iterations observed decreased significantly from 44 in Base-Case to 22 in Case-01.

C. Examples of 3D Problem

We next ran the developed model with a 3D homogeneous reservoir of $75 \times 75 \times 5$ m³ to examine how gravitational effects could be calculated. A grid system of $10 \times 10 \times 2$ was employed, and horizontal permeability of 1000 md and vertical permeability of 500

md were assigned to all grids. The initial pressure and water distributions are shown in Figures 11(a) and (b), respectively. Other data were identical to those used for the 2D simulation.

The production performance obtained by TESIS is shown in Figures 12(a), (b) together with the results of STARS. The major difference is the onset time of incremental oil production, about 100 days for TESIS against about 125 days for STARS. The difference is caused by the gravity under-ride occurring in this displacement process. TESIS modeled this gravity effect in more detail compared with STARS as shown in Figure 13. A significant gravity tongue was developed in the lower layer (see Fig. 13 (b)) which led to early breakthrough of the injected hot water and caused higher production. This is also the reason for the higher oil rates obtained by TESIS than by STARS between 100 and 200 days (see Fig. 12 (a)). Comparisons of the water saturation and pressure profiles shown in Figures 14 and 15 also provide insight into the early increase of oil production. The final water saturation distribution in the upper layer remained low, particularly in regions with less contact with the injected fluid.

VI. CONCLUSIONS

We derived a mathematical model for heat transport based on the streamline method. A numerical scheme using an operator splitting technique was applied to decouple the convection and the diffusions parts for separate solution. A thermal simulator utilizing sequential solutions of mass and heat flow equations was constructed to test this scheme. Simulations of hot water-flooding in

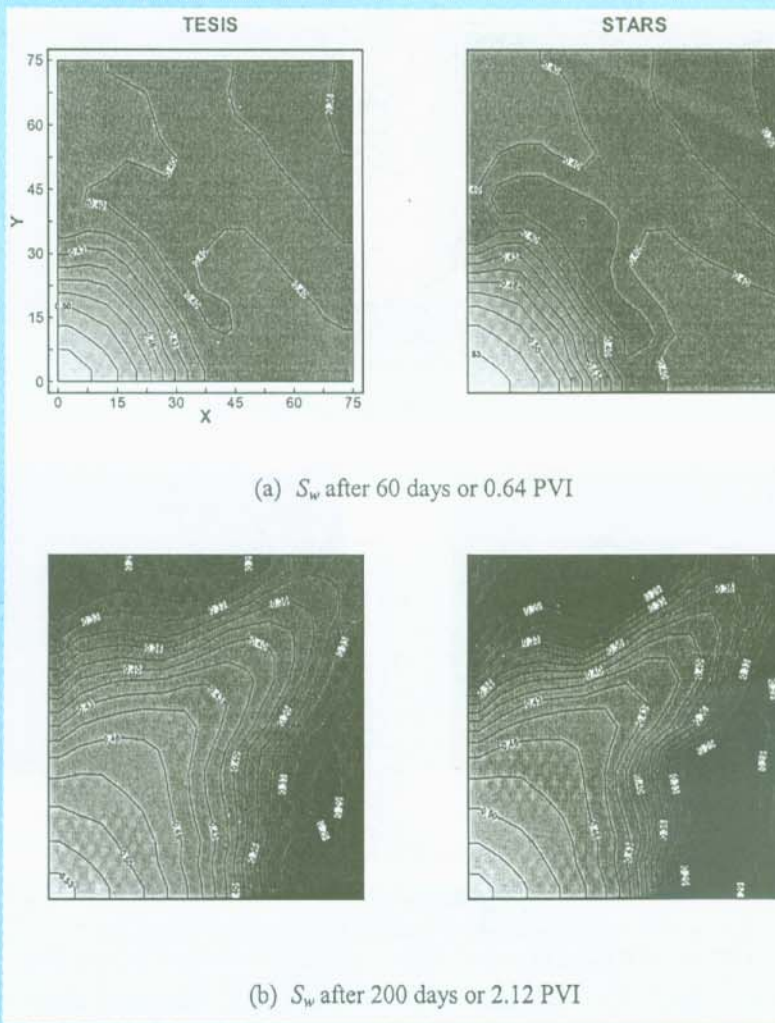


Figure 8
 Comparison of water saturation distributions of 2D Simulation,
 (a) S_w after 60 days or 0.64 PVI, (b) S_w after 200 days or 2.12 PVI

2D and 3D heavy-oil reservoirs were conducted to demonstrate the performance of the developed model.

The results of the 2D simulation compared well with the results of a commercial thermal simulator. Excellent agreements were obtained for the oil recovery performance. The solutions for the heat transport equation were obtained using the irregularly spaced t grid not restricted by CFL conditions. The practical time step was 0.2 to 1.0 times the time step for the pressure solutions. In the 3D simulation, the streamline method demonstrated in more detail how the gravity under-ride mechanism affects the production performance. The test simulations

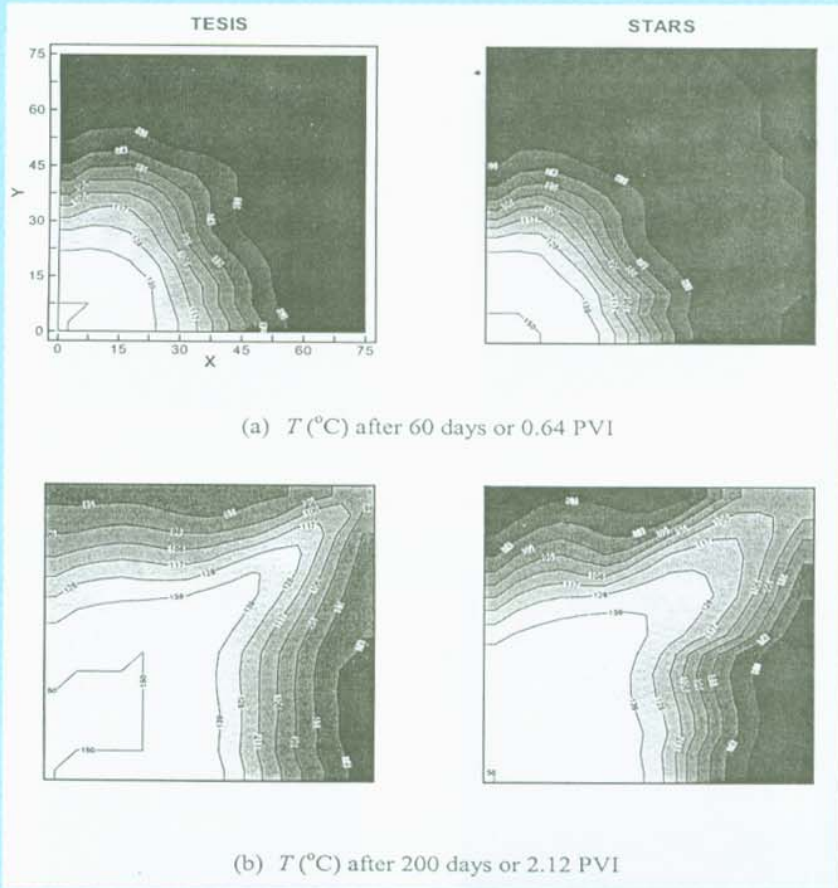


Figure 9
Comparison of temperature distributions of 2D Simulation,
(a) $T(^{\circ}\text{C})$ after 60 days or 0.64 PVI, (b) $T(^{\circ}\text{C})$ after 200 days or 2.12 PVI

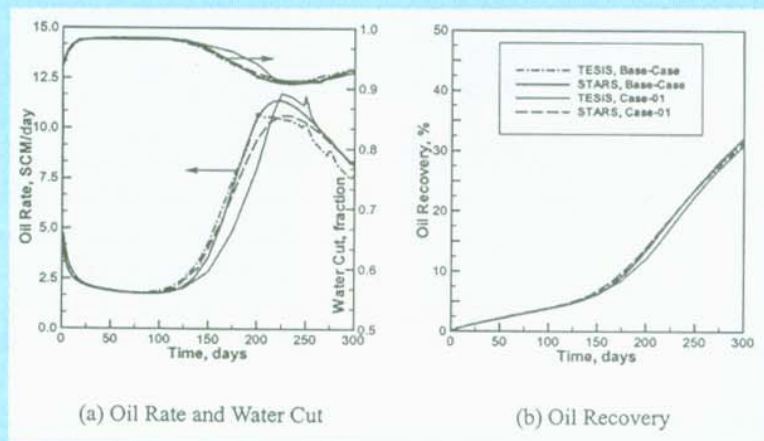


Figure 10
Effects of time step size on production behavior,
(a) Oil rate and water Cut, (b) Oil recovery

demonstrated that the streamline approach could correctly model thermal transport and therefore is a viable alternative to conventional finite-difference models for the heat transport calculation within a thermal simulator. TESIS proved to be no faster than STARS due to inadequate LU decomposition of the matrix solver used in TESIS. However, there is potential for speedup. From the present examples, we found that TESIS required 43 time steps for the 2D model and 40 time steps for the 3D model compared to STARS that required 52 and 63 time steps, respectively. TESIS decoupled the mass and energy conservation equations as opposed to solving them simultaneously in STARS, which results in the smaller time step requirement.

Further work is to implement an appropriate rule of time-step selection for this numerical scheme, to develop an implicit streamline thermal simulator in which saturation and temperature are solved simultaneously along streamlines, and to improve the calculation time.

Acknowledgment

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Nomenclatures

A_x = cross section
 x -di rection [m^2]

- c_r = rock compressibility, [Pa⁻¹]
- C_r = specific heat of rock [kJ/(kg.°C)]
- C_g = specific heat of gas phase [kJ/(kg.°C)]
- D = depth of gridblock from datum [m]
- f_a = fractional flow of phase a [fraction]
- H_a = enthalpy of phase a [kJ/kg]
- k = absolute permeability [m²]
- k_d = time step size for diffusion solution [day]
- K_h = total thermal conductivity [kJ/(m.sec.°C)]
- k_{ra} = relative permeability of phase α [-]
- N_d = number of time step for diffusion solution [-]
- \mathbf{n} = unit vector outward normal to the boundary $\partial\Omega$ [-]
- P_{cwo} = water-oil capillary pressure [Pa]
- p_{ref} = reference pressure [Pa]
- p_a = pressure of phase α [Pa]
- q_a = source or sink flow rate per unit reservoir volume [kg/(day.m³)]
- s = local streamline coordinate [m]
- S_a = saturation of phase a [fraction]
- T = temperature [°C]
- t = time [day]
- \mathbf{u}_a = Darcy velocity of phase a [m/day]

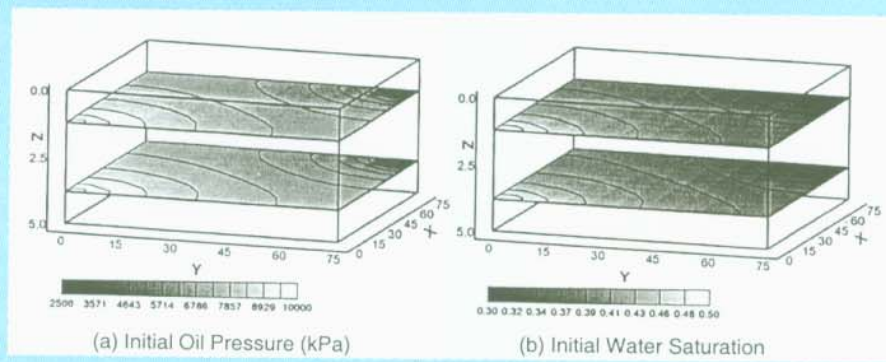


Figure 11
Initial conditions of pressure and water saturation for 3D Case,
(a) Initial oil pressure (kPa), (b) Initial water saturation

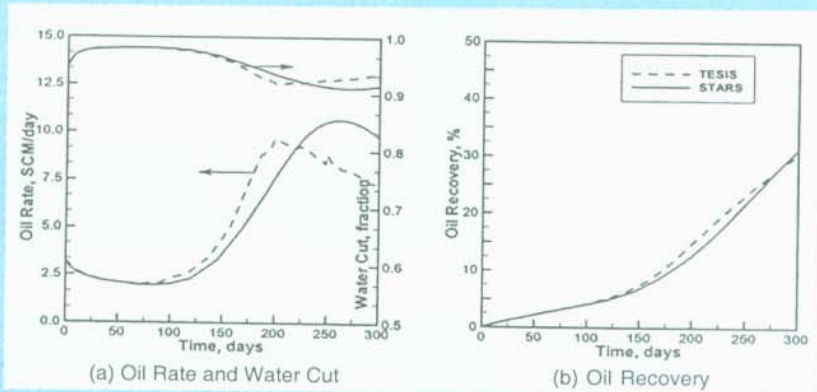


Figure 12
Comparison of production performances of 3D Simulation,
(a) Oil rate and water Cut, (b) Oil recovery

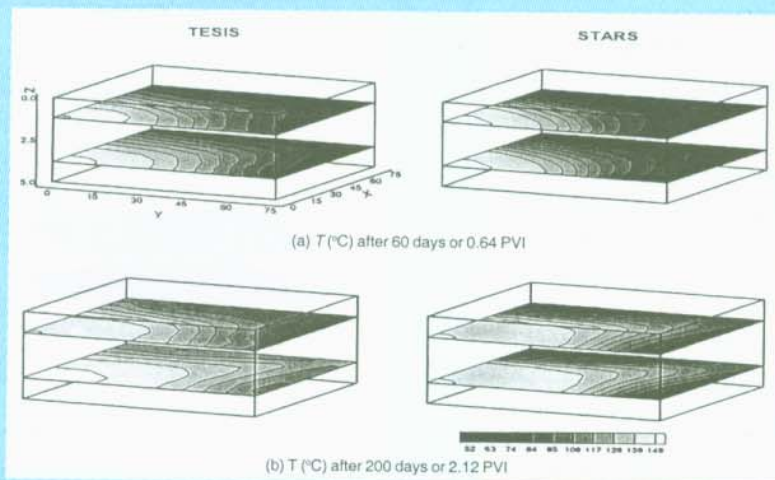


Figure 13
Comparison of temperature distributions of 3D Simulation,
(a) T (°C) after 60 days or 0.64 PVI, (b) T (°C) after 200 days or 2.12 PVI

x = Cartesian domain,
(x, y, z) [-]

<Greeks>

Δt = time step size [day]

Δx = gridblock dimension
in x -direction [m]

Δz = gridblock dimension
in z -direction [m]

ζ = spatial distance coordinate
along stream
line [m]

λ_t = total mobility
[($m^3 \cdot day$)/kg]

λ_α = mobility of phase α
[($m^3 \cdot day$)/kg]

μ_α = viscosity of phase α
[Pa \times day]

Ω = reservoir domain [-]

ρ_r = rock density [kg/ m^3]

ρ_α = density of phase α
[kg/ m^3]

τ = time-of-flight [day]

ϕ = porosity [fraction]

ϕ_{ref} = porosity at reference
pressure [fraction]

<Subscripts>

c = convection

cr = critical

d = diffusion

g = gas

i, j, k = grid block indices

inj = injection

o = oil

sc = surface conditions

t = total

vap = vapor

w = water

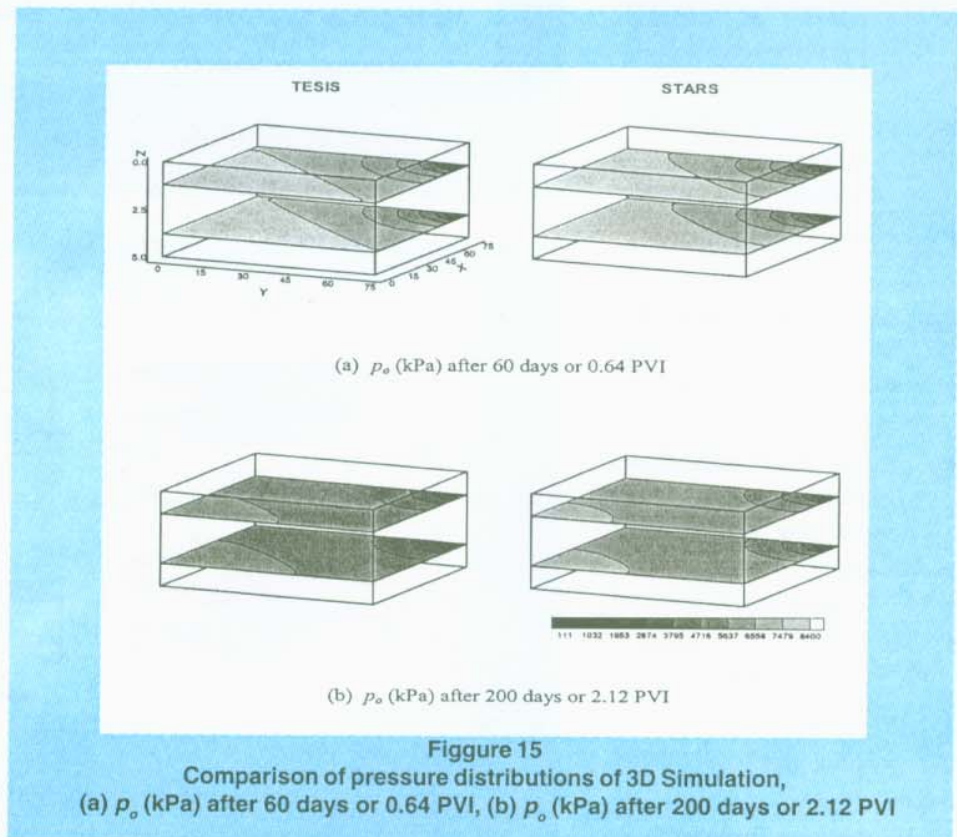
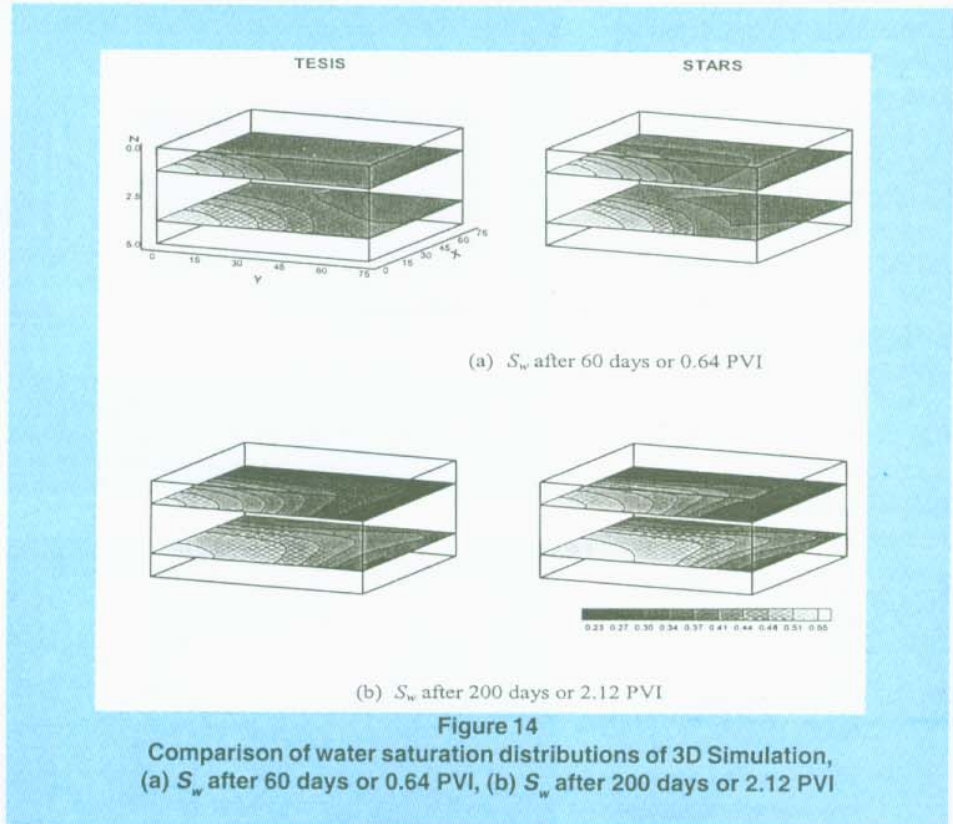
<Superscripts>

k, n = number of time step

o = initial conditions

v = number of Newton it-
erations

w = well.



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