SEISMIC-DERIVED ROCK TRUE RESISTIVITY (R_t) REVISITED. PART I: REFORMULATION OF COMBINED GASSMANN – SHALY SAND MODELS

by

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I. INTRODUCTION

The last decade has observed frantic efforts by geoscientists to extract as much information as possible from seismic data. From the traditional role of establishing subsurface structural geometry, seismic processing and interpretations have evolved into an ever increasing role in providing rock physical properties such as acoustic impedance (AI) and porosity (ϕ). The more common use of 3-D seismic surveys, in both exploration and development stages, have further underlined the role of seismic data as provider of inter-well rock property data.

Further developments in the petrophysics-related seismic interpretation have also shown efforts to extract information related to contents of formation rocks. From the widely acknowledged bright-spot analysis for detecting presence of gas-bearing porous rocks in the last decades of the 20th century to the later efforts to extract information regarding fluid saturation in reservoir. Actually, as early as in mid-1960s have scientists started to investigate the relations between acoustic signals and fluid saturation (e.g King, 1966; Domenico, 1976; Gregory, 1976). However, due to the fact that the then commonly used of 2-D seismic was considered as having insufficient resolution for any practical uses in the field, the efforts remained mainly for academic purposes only.

Rapid developments in technology of 3-D seismic survey and processing, as well as its more common use at present have prompted attentions back to the investigations aimed at extracting fluid saturation information from seismic data. In 1990s have Widarsono and Saptono (1997) started a series of investigation through laboratory measurements and modeling using core samples. This was followed by more works not only at laboratory level but at larger levels of well and field scales (e.g. Widarsono & Saptono, 2000a, 2000b, and 2001; and Widarsono *et al*, 2002a, 2000b). Other investigators (e.g. Furre & Brevik, 2000; Wu, 2000; Zhu *et al*, 2000; and more recently Wu et al, 2005) have also devoted some works to achieving the same goal. Other paths of development have incorporated other supporting techniques such as non-linear regression (e.g. Balch *et al,* 1998) and artificial neural network (e.g. Poupon & Ingram, 1999; Oldenziel *et al*, 2000).

From various investigations using seismic waves as the sole data for fluid saturation extraction, shortcomings were soon felt in the form of 'narrow bands' of acoustic signals (i.e. P-wave velocity, V_p and acoustic impedance, AI) that are influenced by variations in fluid saturation. In other words, V_p and AI are not too influenced by variation in fluid saturation. This reduces the effectiveness of seismic-derived V_p and AI as fluid saturation indicators.

Efforts were then devoted to link V_p and AI to other parameters such as rock true resistivity (R_t), a parameter known to be very sensitive to variation in fluid saturation. Widarsono and Saptono (2003, 2004) provide laboratory verifications and first field trial with some degree of success. However, certain assumptions (i.e. constant/uniform porosity) in the theoretical formulation were still adopted in the above works, which in turn reduced the validity of the resulting formula produced and used.

In this paper, the first part of a three-part work, is devoted to reformulating the combination of Gassmann theory and shaly sand water saturation models of Poupon and Hossin. These are to replace the shale-free Archie model used in the above works, which is considered as invalid for most field uses. With this reformulation, it is hoped that a more robust model/formula of resistivity as a function of acoustic

LEMIGAS SCIENTIFIC CONTRIBUTIONS
VOL. 29. NO. 2, AUGUST 2006: 3 - 9

impedance ($R_t = f(AI)$) is achieved, hence a more reliable resistivity could be extracted from seismic-derived acoustic impedance.

Summarily, the objectives of the works a part of them presented in this paper are

- To establish a model/method to obtain formation rock true resistivity (R_t) from seismic-derived acoustic impedance (AI), and
- To provide correction/modification onto previous works reported in Widarsono & Saptono (2003, 2004).

II. GASSMANN ACOUSTIC VELOCITY MODEL

For any elastic medium, the mechanical response of the medium towards disturbance follows in the general the Hooke's law of

$$\varepsilon = C \times \sigma$$

where ε, C, and σ are strain (deformation), elastic constant, and stress, respectively. Any response of the elastic medium to any imposed stress (σ) in the form of deformation (ε) is governed by the medium's elastic constant (C). Since acoustic waves are basically mechanical disturbance upon a medium, the compressional wave (P-wave) velocity (V_p) of the propagated acoustic waves is governed by the elastic constants in a way of (Fjaer et al, 1992):

$$V_p^2 = \frac{K_b + \frac{4}{3}\mu_b}{\rho_b}$$

Similar expression is also in place for shear wave (S-wave) velocity (V_s)

$$V_s^2 = \frac{\mu_b}{\rho_b}$$

The elastic constants of K_b, μ_b, and ρ_b are the elastic constants of bulk modulus(compliance or incompressibility), shear modulus (rigidity), and bulk density of the medium, respectively. For a porous and fluid saturated elastic medium, the P-wave equation was modified by Gassmann (1951) into

$$V_p^2 = \frac{P_d + f(K_f)}{\rho_b} \tag{1}$$

where P_d is the P-wave modulus for the rock frame (or dry rock), and $f(K_f)$ is the function of the incompressibility of the fluid in the pore spaces. The P-wave modulus for the dry rock can be expressed, in turn, by

$$P_d = K_d + \frac{4}{3}\mu_b \tag{2}$$

and the function $f(K_f)$ by

$$f(K_f) = K_f \frac{(1 - \frac{K_d}{K_m})^2}{(1 - \frac{K_f}{K_m})\phi + (K_m - K_d)\frac{K_f}{K_m^2}} \tag{3}$$

The subscripts *d*, *f*, and *m* refer to the rock frame (or the dry rock,), fluid, and rock matrix.

For rock containing both water and hydrocarbon, the bulk density is expressed as:

$$\rho_b = \phi \cdot \rho_f + (1 - \phi)\rho_m \tag{4}$$

where:

$$\rho_f = S_w \rho_w + (1 - S_w)\rho_{hc} \tag{5}$$

and the fluid incompressibility, K_f, which is the inverse of compressibility, c_f, is given by:

$$K_f = \frac{1}{c_f} = \frac{1}{S_w c_w + (1 - S_w)c_{hc}} \tag{6}$$

where *S* denotes saturation, and the subscript *hc* refers to hydrocarbon.

Rock frame incompressibility, K_d, in Equation (3), which is the inverse of compressibility of dry rock, c_d, is related to pore volume (PV) compressibility, c_p, by:

$$K_d = \frac{1}{c_d} = \frac{1}{\phi \cdot c_p + c_m} \tag{7}$$

The relation between compressional wave velocity (V_p) and water saturation (S_w) is clearly shown by the Equations 1 through 7. There are two governing variables in the main Equation 1 that are influenced by variation in S_w. Even though the two variables in the Equation 1 are reciprocal in nature but the increase in S_w tends to increase the V_p, especially in oil-water two-phase system.

LEMIGAS SCIENTIFIC CONTRIBUTIONS
VOL. 29. NO. 2, AUGUST 2006: 3 - 9

III. SHALY-SAND WATER SATURATION MODELS

On the other hand, the relation between R_t and S_w is more straightforward. This is true since for brine-saturated clean sedimentary rocks the total electrical conductivity is solely governed by the amount of the brine within the pore system. The electric current simply flows through the tortuous pore system that is filled continuously by the brine and completely ignores the non-conductive hydrocarbon fraction and rock matrix.

The relationship is best expressed by the classical Archie formula (Archie, 1942) of

$$S_w^{\;n} = \frac{a}{\phi^m}\frac{R_w}{R_t}$$

where n, a, ϕ, m, and R_w are respectively saturation exponent, twistedness degree (tortuosity) of the rock pore system, porosity, cementation factor (hardness), and brine resistivity. The Archie model above clearly shows the direct influence of variation in S_w to R_t.

The Archie equation is the first water saturation model, and after its establishment in 1942 it has been very widely used in log interpretation throughout the world. However, since the model is valid only for shale-free formation rocks only, it has since then undergone various modifications in order to accommodate presence of shale in its various forms. Dozens of models have been established since then (Bassiouni, 1994).

For the purpose of the works presented in this paper two shaly-sand models, instead of Archie model as used in Widarsono & Saptono (2003, 2004), Poupon and Hossin models have been chosen. The reason for the choice of the two models are their relative simple form and the fact that the two models were derived to accommodate presence of laminated shale (Modified Simandoux) and dispersed shale (Hossin).

The Poupon model (Poupon et al, 1954) is expressed in the form of

$$\frac{1}{S_w^{\;0.5n}} = R_t\left\{\frac{V_{sh}^{\;d}}{\sqrt{R_{sh}}} + \frac{\phi^{0.5m}}{\sqrt{aR_w}}\right\} \tag{8}$$

while the Hossin model (taken from Dresser Atlas, 1982) is expressed in the form of

$$S_w^{\;n} = \frac{aR_w}{\phi^m}\left(\frac{1}{R_t} - \frac{V_{sh}^{\;2}}{R_{sh}}\right) \tag{9}$$

with

S_w = water saturation,
n = saturation exponent,
R_t = rock true resistivity,
V_{sh} = shale fraction,
R_{sh} = shale resistivity,
fð = porosity,
m = cementation factor,
a = tortuosity, and
R_w = formation water resistivity

Note that the two models would be back to the form of Archie model in the case of shale-free ($V_{sh} = 0$) formation rocks. The models in Equations 1 and 2 will introduce R_t into the Gassmann acoustic velocity model.

IV. FORMULATION OF RESISTIVITY FUNCTION

As commonly acknowledged, variations in seismic data as accepted, processed, and interpreted at the surface are due to presence of various reflectors present in the earth crust. These reflectors essentialy reflect differences in properties of the rock layers, including voids and fluids within them, that form the earth crust. The most practical and commonly used term taken to represent these properties of a rock type is the acoustic impedance

$$AI = V_p\rho_b \tag{10}$$

and through the use of Equations 1 and 2, the acoustic impedance can be expressed as

$$AI^2 = \left(K_d + f(K_f) + \frac{4}{3}\mu_b\right)\rho_b$$

If this expression can be simplified through defining the P-wave elastic constant for porous media as að (arbitrary notation)

$$\alpha = K_d + f(K_f) + \frac{4}{3}\mu_b,$$

or

$$\alpha = P_d + f(K_f) \tag{11}$$

LEMIGAS SCIENTIFIC CONTRIBUTIONS
VOL. 29. NO. 2, AUGUST 2006: 3 - 9

if Equation 2 is used, then Equation 10 can be expressed as

$$AI^2 = \alpha\rho_b \qquad (12)$$

We leave Equation 12 as it is and go back to the *f(K)* function in Equation 3. If the function is expressed in term of compressibilities (c = 1/K) and following the Equations 4 through 7, then Equation 3 is presented in the form of

$$f(K_f) = \frac{\left(1-\frac{c_m}{c_d}\right)^2}{(S_w c_w + c_{hc} - S_w c_{hc} - c_m)\phi + \left(c_m - \frac{c_m^{\ 2}}{c_d}\right)} \qquad (13)$$

where subscripts of w, hc, m, and d represent water, hydrocarbon, matrix, and dry condition, respectively. If Equation 13 is simplified following

$$L = \left(1-\frac{c_m}{c_d}\right)^2,$$

$$M = \phi(c_w - c_{hc}),$$

$$N = \phi(c_{hc} - c_m),$$

$$P = \left(c_m - \frac{c_m^{\ 2}}{c_d}\right), \text{ and}$$

$$Q = N + P$$

then it takes the form of

$$f(K_f) = \frac{L}{S_w M + Q} \qquad (14)$$

As the *f(K)* function has been simplified, Equation 11 can then be expressed as

$$\alpha = P_d + \left(\frac{L}{S_w M + Q}\right) \qquad (15)$$

So far the notation aδ has been expanded to fuly contain the Gassmann model in it. Then if the bulk density (rδ$_b$) in Equation 4 is expanded to accommodate presence of shale in the reservoir rocks (assumed as structural shale) of

$$\rho_b = \phi\rho_f + V_{sh}\rho_{sh} + (1 - \phi - V_{sh})\rho_m \qquad (16)$$

then AI in Equation 12 can be expressed as

$$AI^2 = \alpha\left[\phi\rho_f + V_{sh}\rho_{sh} + (1 - \phi - V_{sh})\rho_m\right],$$

and therefore

$$\alpha = \frac{AI^2}{S_w\phi(\rho_w - \rho_{hc}) + \phi(\rho_{hc} - \rho_m) + V_{sh}(\rho_{sh} - \rho_m)} \qquad (17)$$

Again, for the sake of simplication in the formulation, Equation 17 is better expressed in a simpler form following

$$A = AI^2,$$

$$B = \phi(\rho_w - \rho_{hc}),$$

$$C = \phi(\rho_{hc} - \rho_m),$$

$$D = V_{sh}(\rho_{sh} - \rho_m) + \rho_m, \text{ and}$$

$$E = C + D$$

into

$$\alpha = \frac{A}{BS_w + E} \qquad (18)$$

At this point, we have two equations representing α, Equations 15 and 18. By substituting the two equations, and taking

$$F = P_d$$

then

$$\alpha = \frac{A}{BS_w + E} = F + \left(\frac{L}{S_w M + Q}\right)$$

or

$$\frac{A}{BS_w + E} = \frac{F(MS_w + Q) + L}{MS_w + Q}$$

to yield

$$(BFM)S_w^2 + (BFQ + BL + EFM - AM)S_w + (EFQ + EL - AM) = 0 \qquad (19)$$

which is essentially analogous to the form of

$$HS_w^{\ 2} + IS_w + J = 0 \qquad (20)$$

if

$$H = BFM,$$

$$I = BFQ + BL + EFM - AM, \text{ and}$$

$$J = EFQ + EL - AM$$

Then, this kind of quadratic expression is traditionally solved through two S_w roots of

$$S_w = \frac{-I \pm \sqrt{I^2 - 4HJ}}{2H} \qquad (21)$$

But since water saturation (S_w) is always positive between 0 an 1, then the positive root of Equation 21 is taken as the solution.

As shown by Equation 21, the Gassmann model has successfully been rearranged to take the form of water saturation as function of all variables forming the Gassmann model.

It is now the turn to underline that water saturations in any mathematical expressions should have the same general meaning physically, which is basically 'a part of a rock's pore space that is occupied by formation water'. Therefore, it is fundamentally correct to state that

$$S_{w(petrophysics)} = S_{w(Gassmann)} \qquad (22)$$

where $S_{w(petrophysics)}$ is S_w in any water saturation models commonly known in log analysis (e.g. Modified Simandoux and Hossin models) and $S_{w(Gassmann)}$ is S_w in the Gassmann acoustic velocity model (as expressed in Equation 21).

A. *Solution for laminated shale*

Using Poupon model (Equation 8), the expression in Equation 22 is solved through

$$\left[\frac{aR}{\phi(1-V)}\left(\frac{1}{R} - \frac{V}{R}\right)\right]^{\frac{1}{n}} = \frac{-I + \sqrt{I^2 - 4HJ}}{2H} \qquad (23)$$

Following

$$Y = \frac{aR_w}{\phi^m(1-V_{sh})} \qquad \text{and}$$

$$Z = \frac{V_{sh}}{R_{sh}}$$

then Equation 23 becomes

$$\left[Y\left(\frac{1}{R_t} - Z\right)\right]^{\frac{1}{n}} = \frac{-I + \sqrt{I^2 - 4HJ}}{2H}$$

and through some mathematical arrangements has become

$$R_t = \frac{\left(2HY^{\frac{1}{n}}\right)^n}{\left(-I + \sqrt{I^2 - 4HJ}\right)^n + Z\left(2HY^{\frac{1}{n}}\right)^n} \qquad (24)$$

By transforming Equation 24 back to the form of Poupon model it becomes

$$\frac{1}{R_t} = \frac{V_{sh}}{R_{sh}} + \frac{\left(-I + \sqrt{I^2 + 4HJ}\right)^n}{\frac{4aR_w}{\phi^m(1-V_{sh})}T^n} \qquad (25)$$

This Equation 25 is essentially the solution of $R_t = f(AI)$ for the case of formation rocks containing laminated shale. The variable of AI is contained in variables of I and J (i.e. I and J are functions of AI).

B. *Solution for dispersed shale*

Using Hossin model (Equation 9), the expression in Equation 22 is solved through

$$\left[\frac{aR_w}{\phi^m}\left(\frac{1}{R_t} - \frac{V_{sh}^2}{R_{sh}}\right)\right]^{\frac{1}{n}} = \frac{-I + \sqrt{I^2 - 4HJ}}{2H} \qquad (26)$$

Following the same treatment as in the case of laminated shale, Equation 26 becomes

$$\frac{1}{R_t} = \frac{V_{sh}^2}{R_{sh}} + \frac{\left(-I + \sqrt{I^2 + 4HJ}\right)^n}{\frac{4aR_w}{\phi^m}T^n} \qquad (27)$$

Like in the case of laminated shale, variable of AI is contained in variables I and J.

V. DISCUSSION

The two final equations of Equations 25 and 27 have essentially provide us with means to estimate formation rock resistivity with acoustic impedance as the main input. In practice, the two equations may be used in cases, which are characterized by either predominantly laminated-shale or dispersed shale presence. Provided that the paper has shown the derivations performed to yield the two equations, other water saturation models can be used straight away starting from Equation 22 to yield different resistivity functions when the two saturation models used in this work are considered as inappropriate for certain field cases.

The potential problem in the use of Equations 25 and 27 are the various input parameter that have to be acquired. This may lead into difficulties in the application aspect. However, regardless the practical aspect of the resistivity models (practical applications of the two equations will be presented in Part III of this work) they are theoretically and fundamentally correct. Therefore, the models should be taken as reference for possible further development in the quest for establishing a method to derived water saturation from seismic data.

VI. CONCLUSIONS

From the theoretical works presented in this paper, a set of main conclusions have been obtained:

- A theoretical foundation for estimating formation true resistivity data from seismic data has been established.
- The reformulation of the resistivity function has managed to avoid the assumption of constant porosity previously adopted in past studies (i.e. Widarsono & Saptono, 2003; 2004).
- Despite the complicated requirement of supporting input data, the resistivity models formulated in this work are theoretically and fundamentally correct.
- The model derivation has shown that water saturation models used in the formulation of resistivity functions can interchangeable and replaceable easily.
- With the success of the formulation, further developments can be carried out through the use of more sound theory, such as Biot – Gassmann theory, for a more realistic theoretical basis or through the use of simpler models for a more practical use.

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