• A COMPUTER PROGRAM FOR COMPUTING SISMIC WAVE REFLECTION AND TRANSMISSION COEFFICIENTS OF A SINGLE INTERFACE

by

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ABSTRACT

Reflection and transmission properties of a single interface form the basic step in understanding and solving many problems of seismic waves propagation in multilayered media. The computational procedure of these coefficients is an essential element for numerical modeling or simulation purposes.

A computer program for computing seismic wave reflection and transmission coefficients of a single interface has been effectively performed using the Gauss-Jordan elimination technique which is operated on a 4x 4 system of simultaneous linear equations with complex elements. The simultaneous equations have been arranged in matrix from. Elements of the matrix are obtained from the application of the boundary conditions to the solution of the seismic wave equations. The reflection and transmission coefficients for the compressional and shear waves constitute the unknowns of the simultaneous linear equations.

Experiments using the program demonstrate the efficiency and the precision of this procedure.

I. INTRODUCTION

Reflectivity functions are valuable diagnostic parameters in seismic studies. The physical properties of the subsurface rocks can be inferred from the reflectivity calculations. For example, porous zones exhibit significantly stronger compressional reflection functions than do non-porous zones. Within the porous interval more reflection events are generated than in the tight zones.

Transmitivity also contain valuable information, particularly information describing the seismic wave attenuation and dispersion occuring in the sedimentary layers.

Thus, information on the seismic reflection and transmission properties of the subsurface rocks can be used for exploration as well as production purposes. A numerical modeling or simulation system is usually helpful to understand these properties.

It is essential for the numerical modeling/simulation purposes that the physics of the seismic reflection and transmission coefficients be understood, from which a correct mathematical formula can be derived.

In this paper, a computer program for computing reflection/transmission coefficients of a single interface is presented. These coefficients are functions of the wave incident angle and elastic properties of the layers. An incident compressional wave as well as shear wave can be handled. The mode conversions are also taken into account.

II. REFLECTION AND TRANSMISSION COEFFICIENTS OF A SINGLE INTERFACE

The reflection and transmission properties of a single interface is a fundamental concept in seismology. It forms the basic step in understanding and solving many problems of seismic waves propagation in heterogeneous media.

The reflection/transmission coefficients of a single elastic discontinuity as a function of the angle of incidence have been formulated by numerous authors in several different forms (for example Knott, 1899; Zoeppritz, 1919; Muskat and Meres, 1940; Guttenberg, 1944; Ewing et.al., 1957; Brekhovskhikh, 1960; Koefoed, 1962; Tooley et.al., 1965; Cerveny and Ravindra, 1976; Pilant, 1979; Aki and Richards, 1980; Ben Menahem and Singh, 1981).

In this paper we restate Pilant's matrix formulation and outline briefly the basic concept from which this formulation can be derived. We choose a matrix formulation rather than an explicit expression (see for example Ewing et.al., 1957; Cerveny and Ravindra, 1971; Cerveny et.al., 1977) since a matrix formulation enables us to express several simultaneous phenomena in a clear and compact form. The matrices have been arranged in such a way that all elements are expressed in terms of the angle of the incident wave (cf. Frasier, 1970; Aki and Richards, 1980; Ben Menachem and Singh, 1981). This formulation facilitates the computational effort and allows generalisation to the more complex multilayered problem. A crucial typographical error in Pilant's formulation for an incident S wave is corrected in equations (17).

The model assumed consists of two half spaces separated by a horizontal interface. The elastic properties (expressed in terms of P and S wave velocities and densities) above and below this interface are different. A plane wave is travelling upward from the lower medium. This wave produces a stress gradient in the surrounding media which causes particle displacement from the initial/equilibrium position. Considering the infinitesimal stress-strain relationship in an homogeneous, isotropic and perfectly elastic medium, the equation describing the motion of particles can be written as a compact vector notation (see for example Kolsky, 1963; Achenbach, 1973).

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\nabla\nabla.\vec{u} = \rho\ddot{\vec{u}} \qquad (1)$$

where

λ and μ are elastic constants
∇ is the Laplace operator
$\vec{u}$ is the particle displacement.

Equation(1) describes particle motion in term of vector displacement. It corresponds to two types of wave which propagate simultaneously through the medium. It can be decomposed into two types of wave equation in terms of P and S wave displacement potentials.

$$\nabla^2 \phi = \frac{1}{\alpha^2}\frac{\partial^2 \phi}{\partial t^2} \qquad (2)$$

$$\nabla^2 \psi = \frac{1}{\beta^2}\frac{\partial^2 \psi}{\partial t^2} \qquad (3)$$

where

ϕ is the P wave displacement potential
ψ is the S wave displacement potential
α is the P wave velocity $= \sqrt{(\lambda + 2\mu)/\rho}$
β is the S wave velocity $= \sqrt{\mu/\rho}$

The displacement potentials are related to the vector displacement by the equation

$$\vec{u} = \nabla\phi + \nabla \times \vec{\psi} \qquad (4)$$

This decomposition explains that we can treat P and S waves as being decoupled, although once they impinge on an interface, there will be conversion to the other wave type in both reflection and transmission.

Our interest is not with the wave equation itself, but rather with its solution. So far we are still dealing with an homogeneous equation since we have ignored the source of the wavefield. The general solution of this type of wave equation is well known as d'Alembert's solution and can be simplified in the form (after considering radiation conditions).

$$\phi = A_1 f_1 (\vec{k_\alpha}.\vec{x} - \omega t) \qquad (5)$$

$$\psi = A_2 f_2 (\vec{k_\beta}.\vec{x} - \omega t) \qquad (6)$$

where

$\vec{k_\alpha}$ is the wavenumber for the P wave
$\vec{k_\beta}$ is the wavenumber for the S wave
$\vec{x}$ is the distance

Another assumption imposed in the model is that both half spaces are in welded contact at the interface. This guarantees the continuity of the normal and tangential stress and displacement across that interface. In Cartesian coordinates, the normal and tangential stress can be expressed in terms of the displacement potentials as

$$t_{zz} = \lambda \nabla^2 \phi + 2\mu \left[\frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{\partial x \, \partial z} \right] \qquad (7)$$

$$t_{xz} = \mu \left[2 \frac{\partial^2 \phi}{\partial x \partial z} + \left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right) \right] \qquad (8)$$

where

z is the depth and is taken to be positive if is downward and λ , μ , ϕ , ψ , x are defined above.

The normal and tangential displacement in Cartesian coordinates can be written as

$$\vec{w} = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \qquad (9)$$

$$\vec{u} = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} \qquad (10)$$

III. INCIDENT P WAVE

Suppose a plane compressional wave impinges on an interface with an incidence angle θ_{p_1} (Fig. 1). In rotated coordinates the d'Alembert's solution can be written in the form :

For the incident wave

$$\psi_0 = 1 . e^{jk_{\alpha_1} (x \sin \theta_{p_1} - z \cos \theta_{p_1})} \qquad (11)$$

where

k_{α_1} is the horizontal wave number of the P wave in the first medium. The harmonic factor, $e^{j\omega t}$, has been omitted for convenience.

For Reflection/Transmission we have solutions of the form :

$$\phi_1 = r_{PP} e^{jk_{\ 1} (x \sin \theta_{p_1} + z \cos \theta_{p_1})}$$

$$\psi_1 = r_{PS} e^{jk_{\beta_1} (x \sin \theta_{s_1} + z \cos \theta_{s_1})}$$

$$\phi_2 = t_{PP} e^{jk_{\alpha_2} (x \sin \theta_{p_2} - z \cos \theta_{p_2})} \qquad (12)$$

$$\psi_2 = t_{PS} e^{jk_{\beta_2} (x \sin \theta_{s_2} - z \cos \theta_{s_2})}$$

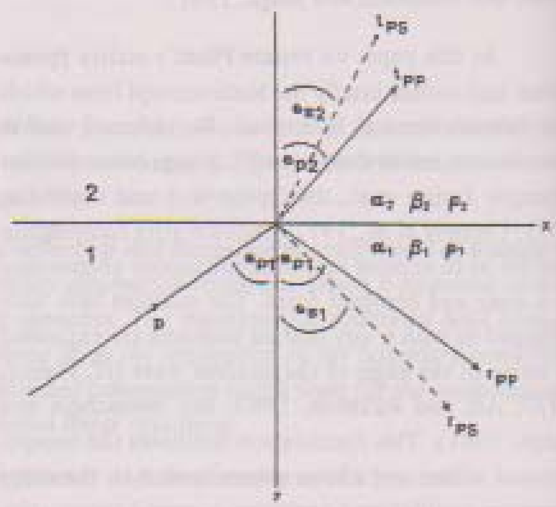

Figure 1

Reflection and transmission at a single interface due to an incident plane compressional wave

where

k_{β_1} is the horizontal wave number of the S medium = ω / β_1

k_{β_2} = ω / β_2, ibid second medium

k_{α_1} was previously defined

k_{α_2} = ω / α_2

Note that r_{PP}, r_{PS}, t_{PP} and t_{PS} are defined with respect to displacement potentials.

The wavefields given by equation (11) and (12) must satisfy the boundary condition viz. continuity of both normal and tangential stress (equations 7 and 8) and displacements (equations 9 and 10) across the interface. This leads to the following simultaneous linear equations which can be written in matrix forms as :

$$\begin{bmatrix} a\sin\theta_{P1} & -(1-a^2\sin^2\theta_{P1})^{1/2} & -a\sin\theta_{P1} & -(b^2-a^2\sin^2\theta_{P1})^{1/2} \\ \cos\theta_{P1} & a\sin\theta_{P1} & (c^2-a^2\sin^2\theta_{P1})^{1/2} & -a\sin\theta_{P1} \\ 2a^2\sin^2\theta_{P1}-1 & -2a\sin\theta_{P1}(1-a^2\sin^2\theta_{P1})^{1/2} & \dfrac{-2a^2\sin^2\theta_{P1}+b^2}{b^2d} & \dfrac{-2a\sin\theta_{P1}(b^2-a^2\sin^2\theta_{P1})^{1/2}}{b^2d} \\ -2a^2\sin\theta_{P1}\cos\theta_{P1} & -2a^2\sin^2\theta_{P1}+1 & \dfrac{-2a\sin\theta_{P1}(c^2-a^2\sin^2\theta_{P1})^{1/2}}{b^2d} & \dfrac{2a^2\sin^2\theta_{P1}-b^2}{b^2d} \end{bmatrix} \begin{bmatrix} r_{PP} \\ r_{PS} \\ t_{PP} \\ t_{PS} \end{bmatrix} = \begin{bmatrix} -a\sin\theta_{P1} \\ a\cos\theta_{P1} \\ -2a^2\sin^2\theta_{P1}+1 \\ -2a^2\sin\theta_{P1}\cos\theta_{P1} \end{bmatrix} \quad (13)$$

The solution of equations (13) yields the reflection and transmission coefficients as a function of angle of incidence due to an incident P wave at a single interface. In this model the elastic properties of the layers are represented by P wave velocity, S wave velocity and density. For the normal incidence case ($\theta_{p1} = 0$), equations (12) reduce to the well known formula.

$$r = \frac{\rho_2\alpha_2 - \rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1}$$

$$t = \frac{2\rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1} \quad (14)$$

The increase of θ_{p1} may cause $(c^2 - a^2\sin^2\theta_{p1})^{1/2}$ and $(b^2 - a^2\sin^2\theta_{p1})^{1/2}$ in equation (13) to become imaginary, which yields the complex reflection and transmission coefficients. This signifies a phase shift between the reflected and the transmitted waves relative to the incident wave. The limit where the coefficient changes from a real number to a complex number is the critical angle.

IV. INCIDENT SV WAVE

For an incident SV wave having an incidence angle of θ_{s_1} (Fig. 2) the d'Alembert's solution in a rotated coordinate system has the form :

$$\psi_0 = 1 . e^{jk\beta_1(x\sin\theta_{s_1} - z\cos\theta_{s_1})} \quad (15)$$

Again, the harmonic factor $e^{j\omega t}$ has been omitted for convenience.

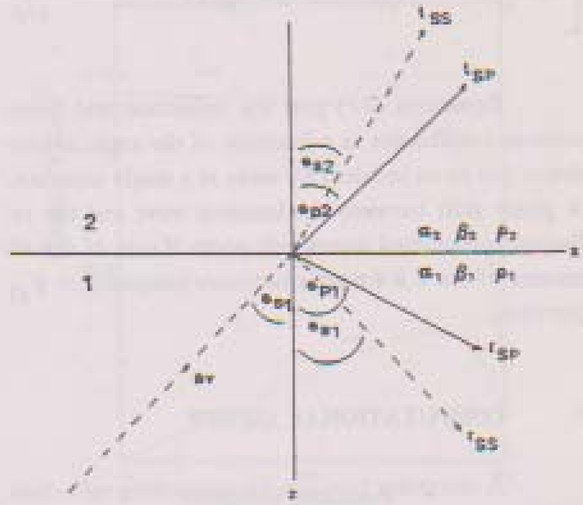

Figure 2
Reflection and transmission at a single interface due to an incident plane SV wave

Upon reflection and transmission we have the solution of the form

$$\begin{aligned} \psi_1 &= r_{SS}\, e^{jk\beta_1(x\sin\theta_{s_1} + z\cos\theta_{s_1})} \\ \phi_1 &= r_{SP}\, e^{jk\beta_1(x\sin\theta_{p_1} + z\cos\theta_{p_1})} \\ \psi_2 &= t_{SS}\, e^{jk\beta_2(x\sin\theta_{s_2} - z\cos\theta_{s_2})} \\ \phi_2 &= t_{SP}\, e^{jk\beta_2(x\sin\theta_{p_2} - z\cos\theta_{p_2})} \end{aligned} \quad (16)$$

The application of the boundary conditions (equations 7 to 10) leads to the following simultaneous linear equations which can be written in matrix form (see equations 17). This result is taken from Pilant (1979, p137) but it should be noted that there is a typographical error in the fourth row of the third column of Pilant's matrix. The correct formula is given below :

$$\begin{bmatrix} \sin\theta_{s1} & -\cos\theta_{s1} & -\sin\theta_{s1} & -(b^2-\sin^2\theta_{s1})^{1/2} \\ (a^2-\sin^2\theta_{s1})^{1/2} & \sin\theta_{s1} & (c^2-\sin^2\theta_{s1})^{1/2} & -\sin\theta_{s1} \\ 2\sin^2{}_{s1}-1 & -2\sin\theta_{s1}\cos\theta_{s1} & \frac{-2\sin^2\theta_{s1}+b^2}{b^2 d} & \frac{-2\sin{}_{s1}(b^2-\sin^2\theta_{s1})^{1/2}}{b^2 d} \\ -2\sin\theta_{s1}(a^2-\sin^2\theta_{s1})^{1/2} & -2\sin^2\theta_{s1}+1 & \frac{-2\sin\theta_{s1}(c^2-\sin^2\theta_{s1})^{1/2}}{b^2 d} & \frac{2\sin^2\theta_{s1}-b^2}{b^2 d} \end{bmatrix} \begin{bmatrix} r_{SP} \\ r_{SS} \\ t_{SP} \\ t_{SS} \end{bmatrix} = \begin{bmatrix} -\cos{}_{s1} \\ -\sin\theta_{s1} \\ -2\sin\theta_{s1}\cos\theta_{s1} \\ 2\sin^2\theta_{s1}-1 \end{bmatrix} \quad (17)$$

Equations (17) give the reflection and transmission coefficients as a function of the angle of incidence due to an incident SV wave at a single interface. A phase shift between the incident wave and the reflected/transmitted waves will occur if one of the elements of the 4 x 4 matrix becomes imaginary as θ_{s1} increases.

V. COMPUTATIONAL ASPECT

A computer program for computing reflection and transmission coefficients as a function of the angle of incidence due to an incident P or S wave at a horizontal interface has been published by Young and Braile (1976). This program is based on an explicit expression given by Cerveny and Ravindra (1971, p. 63).

We have developed alternative computer programs for the same purpose based on the matrix formulation given by equations (13) and (17).

For an incident P wave the coefficients r_{PP}, r_{PS}, t_{PP} and t_{PS} (equations 13) are the unknowns of a 4 x 4 system of simultaneous equations with complex variables. The solution of these equations can be found directly using the Gauss - Jordan elimination technique. For any angle of incidence from 0^o up to 90^o this procedure can be repeated.

A similar procedure is used to compute r_{SP}, r_{SS}, t_{SP} and t_{SS} (equation 17) for an incident SV wave.

The accuracy of the programs has been confirmed by the energy relationships given in equations (18) and (19) (Pilant, 1979, pp. 131, 137).
As a further check, results have been compared with published examples (e.g. Dobrin, 1976, p. 44).

Fig. 3 illustrates the result for the specified model, for the case of an incident P wave. Diagram (a) corresponds to the P wave incident from above (in the high velocity layer). The coefficients r_{PP}, r_{PS}, t_{PP} and t_{PS} are expressed in terms of the energy ratio (see equation 18). We observe that all curves are smoothly varying, signifying the situation of no critical angle. Diagram (b) is for the case of a P wave incident from below (the lower velocity layer). The criti-

$$\left|r_{PP}\right|^2 + \frac{\tan\theta_{P_1}}{\tan\theta_{s_1}}\left|r_{PS}\right|^2 + \frac{\rho_2}{\rho_1}\frac{\tan\theta_{P_1}}{\tan\theta_{P_2}}\left|t_{PP}\right|^2 + \frac{\rho_2}{\rho_1}\frac{\tan\theta_{P_1}}{\tan\theta_{s_2}}\left|t_{PS}\right|^2 = 1 \quad (18)$$

$$\left|r_{SS}\right|^2 + \frac{\tan\theta_{s_1}}{\tan\theta_{P_1}}\left|r_{SP}\right|^2 + \frac{\rho_2}{\rho_1}\frac{\tan\theta_{s_1}}{\tan\theta_{s_2}}\left|t_{SS}\right|^2 + \frac{\rho_2}{\rho_1}\frac{\tan\theta_{s_1}}{\tan\theta_{s_2}}\left|t_{SP}\right|^2 = 1 \quad (19)$$

cal angle is indicated by the sharp discontinuity in the curves. For angles of incidence beyond the critical value, no P wave is transmitted. The reflected P wave first decreases in amplitude in a smoothly varying manner, then it increases suddenly at the critical point. At the same time the reflected S wave amplitude decreases. The decrease of the reflected P wave beyond the critical angle seams to be compensated by the increase of the reflected and transmitted S wave which pass through a maximum beyond the critical angle.

Model :

$\alpha_1 = 7.0$ $\beta_1 = 4.0$ $\rho_1 = 2.65$

$\alpha_2 = 4.3$ $\beta_2 = 2.6$ $\rho_2 = 2.40$ (velocity in km/sec., density in kg/m^3)

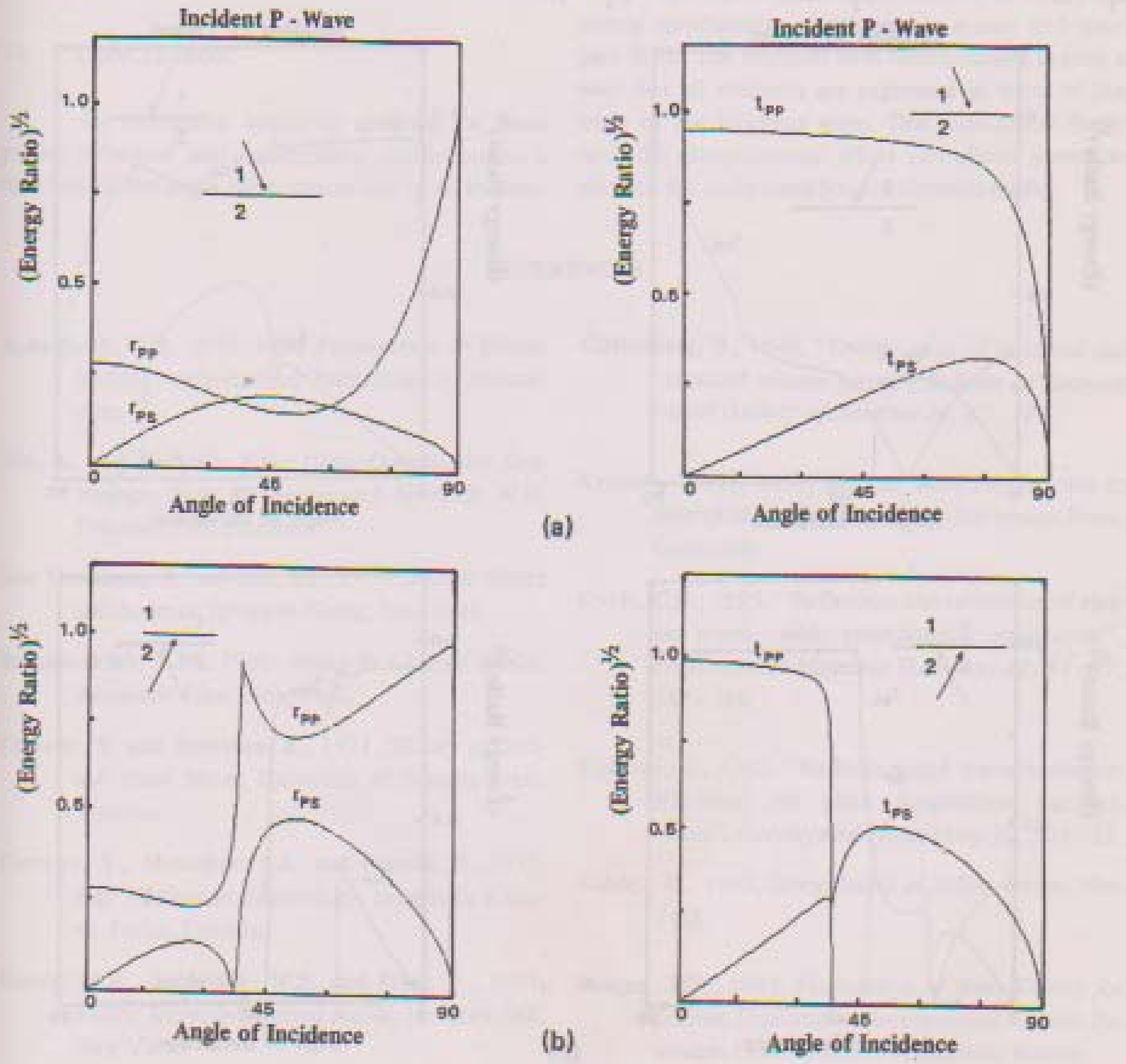

Figure 3

Reflection and transmission coefficients as a function of angle of incidence at a single interface (model given on top). In diagram (a) P wave incident from above. In diagram (b) P wave incident from below

Fig. 4 displays the reflection and transmission coefficients as a function of incidence angle for the case of an incident SV wave. The model parameters are equivalent to those used in the example by Pilant (1979, pp. 132 and 134). Diagram (a) shows r_{SP}, r_{SS}, t_{SP} and t_{SS} when the wave is incident from above (downgoing wave). Again the coefficients are expressed in terms of the square root of the energy ratio (equation 19). An interesting feature to observe is the disappearance of r_{SP} after the first critical angle, followed by a significant increase in t_{SP}. This increase also affects t_{SS}. As t_{SP} disappears, t_{SS} returns to its energy level but decreases again rapidly due to the in-

Model :

$\alpha_1 = 4.6$	$\beta_1 = 2.7$	$\rho_1 = 2.4$
$\alpha_2 = 2.9$	$\beta_2 = 1.6$	$\rho_2 = 1.9$

(velocity in km/sec., density in kg/m^3)

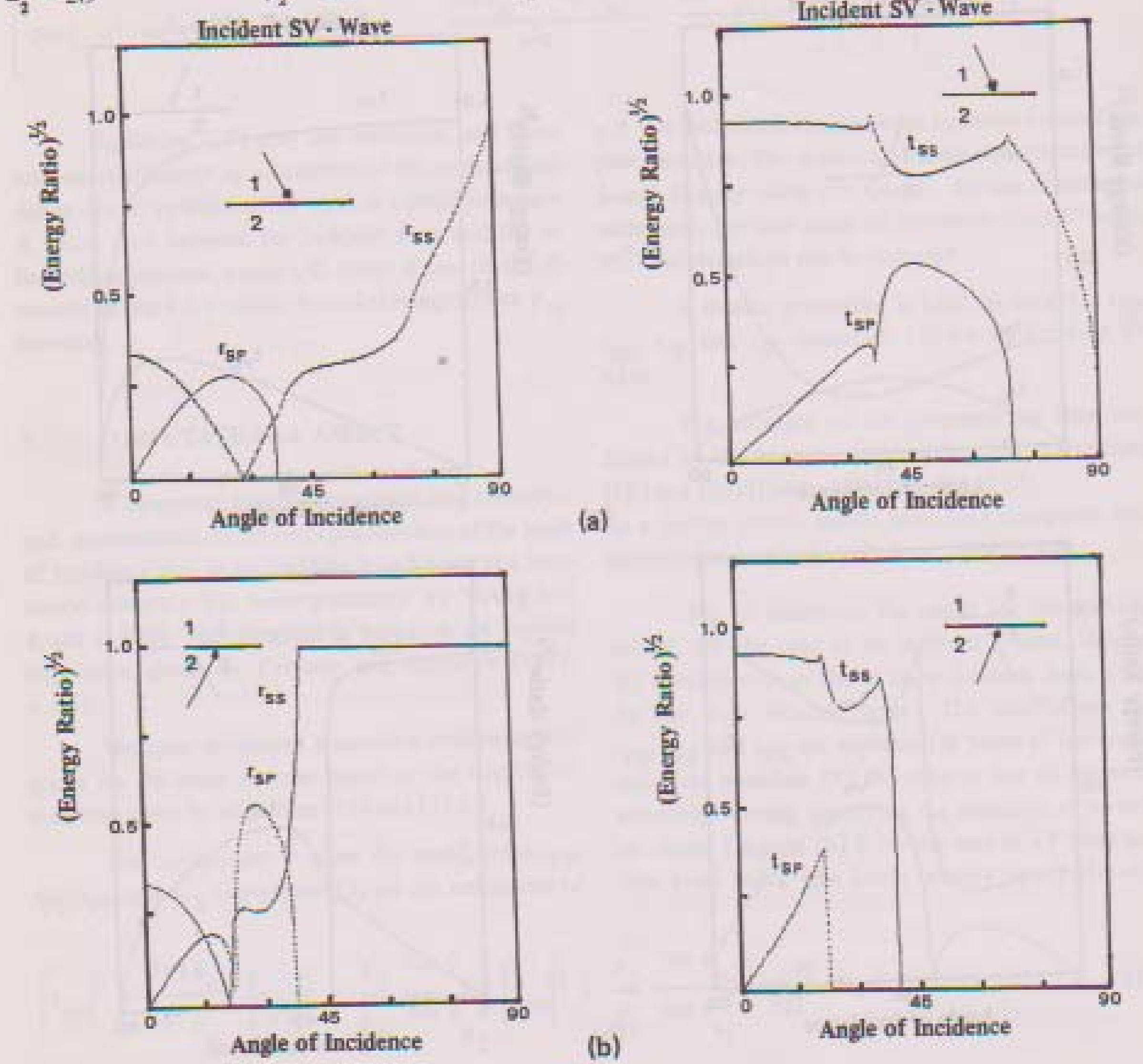

Figure 4

Reflection and transmission coefficients as a function of angle of incidence at a single interface (model given on top). In diagram (a) SV wave incident from above. In diagram (b) SV wave incident from below

crease of r_{SS}. Diagram (b) illustrates the case of the SV wave incident from below (upgoing wave). The two discontinuities in t_{SS} signify the existence of two critical angles for this model. The first critical angle corresponds to the disappearance of t_{SP}, compensated by a significant increase in r_{SP}. The second critical angle represents total reflection of r_{SS} accompanied by the disappearance of t_{SS}.

VI. CONCLUSION

An alternative computer program for computing reflection and transmission coefficients as a function of the angle of incidence due to an incident P or S wave at an horizontal interface has been effectively carried out using matrix formulation. The reflection and transmission coefficients for P wave as well as S wave which form the unknowns of a 4 x 4 systems of simultaneous equations with complex variables can be found directly using the Gauss-Jordan elimination technique. The precision of this method was very high.

The matrix formulation enables us to express several simultaneous phenomena in a clear and compact form. The matrices have been arranged in such a way that all elements are expressed in terms of the angle of the incidents wave. This formulation facilitates the computational effort and allows generalization to the more complex multilayered media.

REFERENCES

Achenbach, J.D. 1973, *Wave Propagation in Elastic Solids*, North-Holland Publishing Co, Amsterdam.

Aki, K. and Richards, P.G., 1980, *Quantitative Seismology, Vol. I, Theory and Methods*, W.H. Freeman, San Fransisco.

Ben Menahem, A. and SIH, S.J., 1981, *Seismic Waves and Sources, Springer-Verlag*, New York.

Brekhovskikh, L.M., 1960, *Waves in Layered Media*, Academic Press, New York.

Cerveny, V. and Ravindra, R., 1971, *Theory of Seismic Head Waves*, University of Toronto Press, Toronto.

Cerveny, V., Molotkov, I.A. and Psencik, I., 1977, *Ray Method in Seismology*, Universita Karlova, Praha, Toronto.

Ewing, M.W., Jardetsky, W.S. and Press, F., 1957, *Elastic Waves in Layered Media*, Mc Graw-Hill, New York.

Frasier, C.W., 1970, "Discrete time solution of plane P-SV Waves in plane layered medium", *Geophysics 35, 197 - 219.*

Guttenberg, B., 1944, "Energy ratio of reflected and refracted seismic waves", *Bulletin of Seismological Society of America 34, 85 - 102.*

Kennett, B.L.N. 1983, *Seismic Wave Propagation in Stratified Media*, Cambridge University Press, Cambridge.

Knott, C.G., 1899, "Reflection and refraction of elastic waves, with seismological application", *Philosophical Magazine (London) 48, 64 - 97, 567 - 569.*

Koefoed, O., 1962, "Reflection and transmission coefficients for plane longitudinal incident waves", *Geophysical Prospecting 10, 304 - 35.*

Kolsky, H., 1963, *Stress Waves in Solids*, Dover, New York.

Morgan, T.R., 1983, *Foundation of Wave Theory for Seismic Exploration*, International Human Resources Development Corporation, Boston.

Muskat, M. and Meres, M.W., 1940, "Reflection and transmission coefficients for plane waves in elastic media", *Geophysics 5, 115 - 148.*

Pilant, W.L., 1979, *Elastic Waves in the Earth*, Elsevier, Amsterdam.

Young, G.B. and Braile, L.W., 1976, "A computer program for the application of Zoeppritz's amplitude equations and Knott's energy equations", *Bulletin of the Seismological Society of America 66, 1881 - 1885.*

APPENDIX

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C
C     EXAMPLE OF TESTING PROGRAM FOR COMPUTING: RPP,RPS,TPP,TPS
C
      DIMENSION R11(100),R12(100),T11(100),T12(100),SUM(100)
      COMPLEX RPP,RPS,TPP,TPS
      COMPLEX SINTP1,COSTP1
      COMPLEX SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,COSTP2
      EPS=1.0E-09
      VP2=21000.
      VP1=12000.
      VS2=12000.
      VS1=8000.
      RHO2=2.65
      RHO1=2.40
      DO 2 I=1,100
    2 SUM(I)=0.0
      DO 1 I=1,100
      GAM=FLOAT(I)*3.14159*90./(100.*180.)
      STP1=SIN(GAM)
      CTP1=COS(GAM)
      SINTP1=CMPLX(STP1,EPS)
      COSTP1=CMPLX(CTP1,EPS)
      WRITE(11,201) SINTP1,COSTP1
  201 FORMAT(4(3X,E16.6))
      CALL ANGLEP(VP1,VP2,VS1,VS2,RHO1,RHO2,GAM,SINTP1,COSTP1,
     1SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,COSTP2)
      CALL CRTPS(VP1,VP2,VS1,VS2,RHO1,RHO2,SINTP1,COSTP1,RPP,RPS,
     1TPP,TPS)
      R1=RPP*CONJG(RPP)
      R121=ABS(R1)
      R11(I)=SQRT(R121)
      R2=RPS*CONJG(RPS)
      T1=TPP*CONJG(TPP)
      T2=TPS*CONJG(TPS)
      SUM2=SINTP1*COSTS1*R2/(COSTP1*SINTS1)
      SUM3=RHO2*SINTP1*COSTP2*T1/(RHO1*COSTP1*SINTP2)
      SUM4=RHO2*SINTP1*COSTS2*T2/(RHO1*COSTP1*SINTS2)
      SUM(I)=SUM(I)+(R1+SUM2+SUM3+SUM4)
      SUM21=ABS(SUM2)
      R12(I)=SQRT(SUM21)
    1 CONTINUE
C
C     R1   = SQUARE OF THE ENERGY FROM RPP
C     R11  = SQUARE ROOT OF THE ENERGY RATIO FROM RPP
C     R12  = SQUARE ROOT OF THE ENERGY RATIO FROM RPS
C     SUM2 = SQUARE OF THE ENERGY RATIO FROM RPS
C     SUM3 = SQUARE OF THE ENERGY RATIO FROM TPP
C     SUM4 = SQUARE OF THE ENERGY RATIO FROM TPS
C     SUM  = TOTAL SQUARE OF THE ENERGY RATIO
C            USED TO CHECK THE CONSERVATION OF ENERGY
C
      WRITE(11,203) (R11(I),R12(I),I=1,100)
C     WRITE(11,204) (SUM(I),I=1,100)
  204 FORMAT(E16.6)
  203 FORMAT(2E16.6)

      STOP
      END
```

```
      SUBROUTINE ANGLEP(VP1,VP2,VS1,VS2,RHO1,RHO2,GAM,SINTP1,COSTP1
     1 ,SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,COSTP2)
      COMPLEX SINTP1,COSTP1,SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,
     2 COSTP2

      EPS=1.ØE-9
      A=VS1/VP1
      B=VS1/VS2
      C=VS1/VP2
      D=RHO1/RHO2

      TETAP1=GAM
      STP1=SIN(TETAP1)
      SINTP1=CMPLX(STP1,EPS)
      CTP1=COS(TETAP1)
      COSTP1=CMPLX(CTP1,EPS)

      STS1=A*STP1
      SINTS1=CMPLX(STS1,EPS)
      CTS11=1.Ø-STS1**2
      IF(CTS11.LT.Ø.Ø) GO TO 5
      CTS12=SQRT(CTS11)
      COSTS1=CMPLX(CTS12,EPS)
      GO TO 6
    5 CTS12=SQRT(ABS(CTS11))
      COSTS1=CMPLX(EPS,CTS12)

    6 STS2=STS1/B
      SINTS2=CMPLX(STS2,EPS)
      CTS21=1.Ø-STS2**2
      IF(CTS21) 1,1Ø,11
   11 CTS2=SQRT(CTS21)
      COSTS2=CMPLX(CTS2,EPS)
      GO TO 2
   1Ø COSTS2=CMPLX(EPS,EPS)
      GO TO 2
    1 CTS2=SQRT(ABS(CTS21))
      COSTS2=CMPLX(EPS,CTS2)

    2 STP2=STS1/C
      SINTP2=CMPLX(STP2,EPS)
      CTP21=1.Ø-STP2**2
      IF(CTP21) 3,3Ø,31
   31 CTP2=SQRT(CTP21)
      COSTP2=CMPLX(CTP2,EPS)
      GO TO 4
   3Ø COSTP2=CMPLX(EPS,EPS)
      GO TO 4
    3 CTP2=SQRT(ABS(CTP21))
      COSTP2=CMPLX(EPS,CTP2)
C
    4 RETURN
      END
```

```
      SUBROUTINE CRTPS(VP1,VP2,VS1,VS2,RHO1,RHO2,SINTP1,COSTP1,RPP,RPS,
     1 TPP,TPS)
C
C     SUBROUTINE FOR COMPUTING REFLECTION AND TRANSMISSION
C     COEFF. FOR A SINGLE HORIZONTAL INTERFACE DUE TO AN
C     INCIDENT P-WAVE
C     WRITTEN BY SUPRAJITNO, GEOPHYSICS-LEMIGAS.
C
      DIMENSION AM(4,5), X(4)
      COMPLEX AM,X,RPP,RPS,TPP,TPS
      COMPLEX SINTP1,COSTP1
C
      A=VS1/VP1
      B=VS1/VS2
      C=VS1/VP2
      D=RHO1/RHO2
C
      EPS=1.ØE-9
      AM(1,1)=A*SINTP1
      AM121=1.Ø-(A*A*SINTP1*CONJG(SINTP1))
      AM12=-SQRT(AM121)
      AM(1,2)=CMPLX(AM12,EPS)
      AM(1,3)=-A*SINTP1
      AM141=B*B-(A*A*SINTP1*CONJG(SINTP1))
      IF(AM141) 1,1ØØ,1Ø1
 1Ø1  AM14=-SQRT(AM141)
      AM(1,4)=CMPLX(AM14,EPS)
      AM(3,4)=2.*A*SINTP1*AM(1,4)/(B*B*D)
      GO TO 2
 1ØØ  AM(1,4)=CMPLX(EPS,EPS)
      AM(3,4)=CMPLX(EPS,EPS)
      GO TO 2
 1    AM141=A*A*SINTP1*CONJG(SINTP1)-(B*B)
      AM14=-SQRT(AM141)
      AM(1,4)=CMPLX(EPS,AM14)
      AM(3,4)=2.*A*SINTP1*AM(1,4)/(B*B*D)
 2    AM(2,1)=A*COSTP1
      AM(2,2)=A*SINTP1
      AM231=C*C-(A*A*SINTP1*CONJG(SINTP1))
      IF(AM231) 3,3ØØ,3Ø1
 3Ø1  AM23=SQRT(AM231)
      AM(2,3)=CMPLX(AM23,EPS)
      AM(4,3)=-2.*A*SINTP1*AM(2,3)/(B*B*D)
      GO TO 4
 3ØØ  AM(2,3)=CMPLX(EPS,EPS)
      AM(4,3)=CMPLX(EPS,EPS)
      GO TO 4
 3    AM231=A*A*SINTP1*CONJG(SINTP1)-(C*C)
      AM23=SQRT(AM231)
      AM(2,3)=CMPLX(EPS,AM23)
      AM(4,3)=-2.*A*SINTP1*AM(2,3)/(B*B*D)
 4    AM(2,4)=-A*SINTP1
      AM31=2.*A*A*SINTP1*CONJG(SINTP1)-1.Ø
      AM(3,1)=CMPLX(AM31,EPS)
      AM322=1.Ø-(A*A*SINTP1*CONJG(SINTP1))
      AM(3,2)=-2.*A*SINTP1*SQRT(AM322)
      AM33=(-2.*A*A*SINTP1*CONJG(SINTP1)+B*B)/(B*B*D)

      AM(3,3)=CMPLX(AM33,EPS)

      AM(4,1)=-2.*A*A*SINTP1*COSTP1
      AM(4,2)=-2.*A*A*SINTP1*CONJG(SINTP1)+1.Ø
      AM44=(2.*A*A*SINTP1*CONJG(SINTP1)-B*B)/(B*B*D)
      AM(4,4)=CMPLX(AM44,EPS)
      AM(1,5)=-A*SINTP1
      AM(2,5)=A*COSTP1
      AM35=-2.*A*A*SINTP1*CONJG(SINTP1)+1.Ø
      AM(3,5)=CMPLX(AM35,EPS)
      AM(4,5)=-2.*A*A*SINTP1*COSTP1
C
      CALL GAUSJR(AM,4,5,X)
      RPP=X(1)
      RPS=X(2)
      TPP=X(3)
      TPS=X(4)
C
      RETURN
      END
```

```
C
C     SUBROUTINE TO SOLVE SIMULTANEOUS LINEAR EQUATIONS WITH
C     COMPLEX ELEMENTS USING THE GAUSS-JORDAN ELIMINATION
C     TECHNIQUE : WRITTEN BY SUPRAJITNO, GEOPHISICS-LEMIGAS
C
      SUBROUTINE GAUSJR(A,N,N1,X)
      DIMENSION A(N,N1),X(N)
      COMPLEX A,X,OP
C     N1 = N+1
      DO 1 IP=1,N
      DO 2 I=1,N
      IF(I.EQ.IP) GO TO 2
      OP=-A(I,IP)/A(IP,IP)
      I1=IP+1
      DO 3 J=IP,N1
      A(I,J)=A(I,J)+OP*A(IP,J)
    3 CONTINUE
    2 CONTINUE
    1 CONTINUE
      DO 4 I=1,N
      X(I)=A(I,N1)/A(I,I)
    4 CONTINUE
      RETURN
      END
```

```
C
C     EXAMPLE OF TESTING PROGRAM FOR COMPUTING: RSF,RSS,TSP AND TSS
C
      DIMENSION R11(200),R12(200),T11(200),T12(200)
      DIMENSION SUM(200)
      COMPLEX RSP,RSS,TSP,TSS
      COMPLEX SINTP1,COSTP1,SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,COSTP2
      PI=3.14159
      VP1=13055.
      VP2=8596.
      VS1=8000.
      VS2=4960.
      RHO1=2.40
      RHO2=1.896
      NLIM=100
      SCALE=90./100.
      DO 2 I=1,NLIM
    2 SUM(I)=0.0
      DO 1 I=1,NLIM
      GAM=SCALE*FLOAT(I)*PI/180.
      CALL ANGLES(VP1,VP2,VS1,VS2,RHO1,RHO2,GAM,SINTP1,COSTP1,
     1SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,COSTP2)
      CALL CRTSP(VP1,VP2,VS1,VS2,RHO1,RHO2,SINTS1,COSTS1,RSP,RSS,
     1TSP,TSS)
      R1=RSP*CONJG(RSP)
      R2=RSS*CONJG(RSS)
      R2=ABS(R2)
      R12(I)=SQRT(R2)
      T1=TSP*CONJG(TSP)
      T2=TSS*CONJG(TSS)
      SUM2=SINTS1*COSTP1*R1/(COSTS1*SINTP1)
      SUM3=RHO2*SINTS1*COSTS2*T2/(RHO1*COSTS1*SINTS2)
      SUM4=RHO2*SINTS1*COSTP2*T1/(RHO1*COSTS1*SINTP2)
      SUM(I)=SUM(I)+(R2+SUM2+SUM3+SUM4)
      SUM2=ABS(SUM2)
      SUM3=ABS(SUM3)
      SUM4=ABS(SUM4)
      R11(I)=SQRT(SUM2)
      T11(I)=SQRT(SUM4)
      T12(I)=SQRT(SUM3)
    1 CONTINUE
      WRITE(10,201) (R11(I),R12(I),I=1,NLIM)
  201 FORMAT(2E16.6)
      STOP
      END
```

```
      SUBROUTINE ANGLES(VP1,VP2,VS1,VS2,RHO1,RHO2,GAM,SINTP1,COSTP1
     1,SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,COSTP2)
      COMPLEX SINTP1,COSTP1,SINTS1,COSTS1,SINTS2,COSTS2,SINTP2,COSTP2

      EPS=1.0E-09
      A=VS1/VP1
      B=VS1/VS2
      C=VS1/VP2
      D=RHO1/RHO2

      TETAS1=GAM
      STS1=SIN(TETAS1)
      SINTS1=CMPLX(STS1,EPS)
      CTS1=COS(TETAS1)
      COSTS1=CMPLX(CTS1,EPS)

      STP1=STS1/A
      SINTP1=CMPLX(STP1,EPS)
      CTP11=1.0-STP1**2
      IF(CTP11) 1,10,11
   11 CTP1=SQRT(CTP11)
      COSTP1=CMPLX(CTP1,EPS)
      GO TO 2
   10 COSTP1=CMPLX(EPS,EPS)
      GO TO 2
    1 CTP1=SQRT(ABS(CTP11))
      COSTP1=CMPLX(EPS,CTP1)

    2 STS2=STS1/B
      SINTS2=CMPLX(STS2,EPS)
      CTS21=1.0-STS2**2
      IF(CTS21) 3,30,31
   31 CTS2=SQRT(CTS21)
      COSTS2=CMPLX(CTS2,EPS)
      GO TO 4
   30 COSTS2=CMPLX(EPS,EPS)
      GO TO 4
    3 CTS2=SQRT(ABS(CTS21))
      COSTS2=CMPLX(EPS,CTS2)

    4 STP2=STS1/C
      SINTP2=CMPLX(STP2,EPS)
      CTP21=1.0-STP2**2
      IF(CTP21) 5,500,501

  501 CTP2=SQRT(CTP21)
      COSTP2=CMPLX(CTP2,EPS)
      GO TO 6
  500 COSTP2=CMPLX(EPS,EPS)
      GO TO 6
    5 CTP2=SQRT(ABS(CTP21))
      COSTP2=CMPLX(EPS,CTP2)

    6 CONTINUE

      RETURN
      END
```

```
      SUBROUTINE CRTSP(VP1,VP2,VS1,VS2,RHO1,RHO2,SINTS1,COSTS1,RSP,RSS,
     1TSP,TSS)
C
C     SUBROUTINE FOR COMPUTING REFLECTION AND TRANSMISSION COEFF.
C     FOR A SINGLE HORIZONTAL INTERFACE DUE TO AN INCIDENT SV-WAVE
C     WRITTEN BY SUPRAJITNO, GEOPHYSICS-LEMIGAS.
C
      DIMENSION  R11(1ØØ),R12(1ØØ),T11(1ØØ),T12(1ØØ),AM(4,5),X(4)
      COMPLEX RSP,RSS,TSP,TSS,AM,X
      COMPLEX SINTS1,COSTS1

      EPS=1.ØE-Ø9
      A=VS1/VP1
      B=VS1/VS2
      C=VS1/VP2
      D=RHO1/RHO2
      AM(1,1)=SINTS1
      AM(1,2)=-COSTS1
      AM(1,3)=-SINTS1
      AM141=B*B-(SINTS1*CONJG(SINTS1))
      IF(AM141) 1,1Ø1,1Ø2
 1Ø2  AM14=-SQRT(AM141)
      AM(1,4)=CMPLX(AM14,EPS)
      AM(3,4)=2.*SINTS1*AM(1,4)/(B*B*D)
      GO TO 2
 1Ø1  AM(1,4)=CMPLX(EPS,EPS)
      AM(3,4)=CMPLX(EPS,EPS)
      GO TO 2
 1    AM141=SINTS1*CONJG(SINTS1)-(B*B)
      AM14=-SQRT(AM141)
      AM(1,4)=CMPLX(EPS,AM14)
      AM(3,4)=2.*SINTS1*AM(1,4)/(B*B*D)
 2    AM211=A*A-(SINTS1*CONJG(SINTS1))
      IF(AM211) 3,3Ø1,3Ø2
 3Ø2  AM21=SQRT(AM211)
      AM(2,1)=CMPLX(AM21,EPS)
      AM(4,1)=-2.*SINTS1*AM(2,1)
      GO TO 4
 3Ø1  AM(2,1)=CMPLX(EPS,EPS)
      AM(4,1)=CMPLX(EPS,EPS)
      GO TO 4
 3    AM211=SINTS1*CONJG(SINTS1)-(A*A)
      AM21=SQRT(AM211)
      AM(2,1)=CMPLX(EPS,AM21)
      AM(4,1)=-2.*SINTS1*AM(2,1)
 4    AM(2,2)=SINTS1
      AM231=C*C-(SINTS1*CONJG(SINTS1))
      IF(AM231) 5,5Ø1,5Ø2
 5Ø2  AM23=SQRT(AM231)
      AM(2,3)=CMPLX(AM23,EPS)
      AM(4,3)=-2.*SINTS1*AM(2,3)/(B*B*D)
      GO TO 6
 5Ø1  AM(2,3)=CMPLX(EPS,EPS)
      AM(4,3)=CMPLX(EPS,EPS)
      GO TO 6
 5    AM231=SINTS1*CONJG(SINTS1)-(C*C)

      AM23=SQRT(AM231)
      AM(2,3)=CMPLX(EPS,AM23)
      AM(4,3)=-2.*SINTS1*AM(2,3)/(B*B*D)
 6    AM(2,4)=-SINTS1
      AM31=2.*SINTS1*CONJG(SINTS1)-1.Ø
      AM(3,1)=CMPLX(AM31,EPS)
      AM(3,2)=-2.*SINTS1*COSTS1
      AM33=(-2.*SINTS1*CONJG(SINTS1)+B*B)/(B*B*D)
      AM(3,3)=CMPLX(AM33,EPS)
C     AM34=(AM341*SQRT(ABS(AM141)))/(B*B*D)
      AM42=-2.*SINTS1*CONJG(SINTS1)+1.Ø
      AM(4,2)=CMPLX(AM42,EPS)
      AM44=(2.*SINTS1*SINTS1-B*B)/(B*B*D)
      AM(4,4)=CMPLX(AM44,EPS)
      AM(1,5)=-COSTS1
      AM(2,5)=-SINTS1
      AM(3,5)=-2.*SINTS1*COSTS1
      AM45=2.*SINTS1*CONJG(SINTS1)-1.Ø
      AM(4,5)=CMPLX(AM45,EPS)
C
      CALL GAUSJR(AM,4,5,X)
C   *
      RSP=X(1)
      RSS=X(2)
      TSP=X(3)
      TSS=X(4)
      RETURN
      END
```