SEPARATION OF UP GOING AND DOWN GOING WAVES IN VSP USING SINGULAR VALUE DECOMPOSITION

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ABSTRACT

The separation of upgoing and downgoing waves in Vertical Seismic Profiling (VSP) is effectively accomplished using the singular value decomposition method. The VSP data can be decomposed into eigenimages which is directly proportional to the singular value.

In the Singular Value Decomposition, the VSP data can be grouped into three successive intervals, i.e., the lowpass component which represents, the downgoing waves, the bandpass component which represents the upgoing waves and the highpass component which represents the noise. The limit between the lowpass and the bandpass (p), the bandpass and the highpass (q) are signified by the drastic change in the singular value plot.

The effectiveness of this method has been clearly demonstrated by processing VSP synthetic examples. The method has also done a very good job in separating upgoing and downgoing waves in VSP in the present of noise as well as in the case where the geophone interval is irregular.

I. INTRODUCTION

The separation of upgoing waves (UGW) and downgoing waves (DGW) in vertical seismic profiling (VSP) record is one of the essential steps in processing VSP data. This is due to the fact that the interference between the UGW and the DGW always occurs in VSP record.

The VSP measurements is illustrates in Figure-1. The DGW carry information from the layers above the receiver, while the UGW carry information from the layers below the receiver. Since reflection absorbs more energy than the transmissions, that the UGW amplitude is always small compared to the DGW amplitude.

The interference between the UGW and the DGW always occurs in VSP record which causes the UGW corrupted by the DGW. In this case, extracting information from the layers below the receiver which is carried by the UGW is impossible, unless the separation between the UGW and the DGW has been done effectively.

To solve the problem several approaches have been proposed. Seeman and Horowicz (1983) introduced an optimum velocity filter designed on the leat-square norm. Gaiser and Di Siena (1982), Suprajitno and Greenhalgh (1985) proposed a modified version of an f–k filter for VSP applications. Kennett and Ireson (1981), Hardage (1983) introduced an alternative method for separating UGW and DGW using the subtraction technique. A medium filter was also proposed for separating UGW and DGW (Bednar, 1982, Evans, 1982; Fitch and Dillon, 1983).

In this paper, a new approach for separating UGW and DGW was tested using synthetic

VSP data from several models. This approach called the Singular Value Decomposition (SVD) is a mathematics by origin. It was recently applied to solve geophysical problem, especially to VSP data (Ursin and Zheng, 1985; Freire, Sergio and Ulrych, 1988).

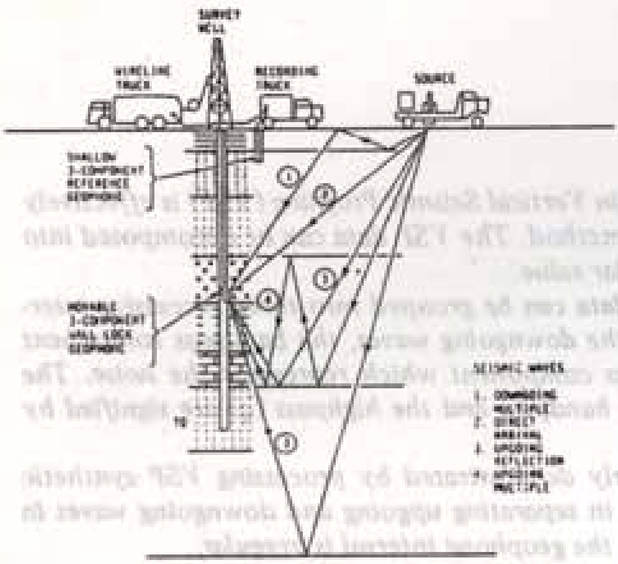

Figure 1. **Wave paths in Vertical Seismic Profiling (Toksoz and Stewart, 1984).**

II. MATHEMATICAL BACKGROUND

The Singular Value Decomposition (SVD) is based a theorem which states that a matrix X (m x m) can be decomposed into the product of three matrices.

$$X = U \Delta V^T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in this case U in a matrix m x m whose columns consist of eigenvectors of a covariant matrix XX^T. This eigenvectors are orthonormal so that $UU^T = U^T U = I$.

V is a matrix m x m whose columns consist of eigenvectors of a covariant matrix $X^T X$. Δ is a matrix m x m whose diagnonal elements consist of singular value of X i.e $\sigma_1, \sigma_2, \sigma_3 \ldots\ldots \sigma_n$.

In matrix algebra, equation (1) can be written as

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & x_{mn} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \cdots & u_{1m} \\ u_{21} & u_{22} \cdots & u_{2m} \\ \vdots & \vdots & \vdots \\ u_{m1} & u_{m2} & u_{mm} \end{bmatrix} *$$

$$\begin{bmatrix} \sigma_1 & 0 \cdots & 0 & 0 \cdots & 0 \\ 0 & \sigma_2 \cdots & 0 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 \cdots & \sigma_m & 0 \cdots & 0 \end{bmatrix} * \begin{bmatrix} v_{11} & v_{21} \cdots & v_{m1} \cdots & v_{n1} \\ v_{12} & v_{22} \cdots & v_{m2} \cdots & v_{n2} \\ v_{13} & v_{23} \cdots & v_{m3} \cdots & v_{n3} \\ \vdots & \vdots & \vdots & \vdots \\ v_{1m} & v_{2m} \cdots & v_{mm} \cdots & v_{nm} \\ v_{1m+1} & v_{2m+1} \cdots & v_{mm+1} \cdots & v_{nm+1} \\ \vdots & \vdots & \vdots & \vdots \\ v_{1n} & v_{2n} \cdots & v_{mn} \cdots & v_{nn} \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & x_{mn} \end{bmatrix} = \begin{bmatrix} \sigma_1 u_{11} & \sigma_2 u_{12} \cdots & \sigma_m u_{1m} & 0 \cdots & 0 \\ \sigma_1 u_{21} & \sigma_2 u_{22} \cdots & \sigma_m u_{2m} & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma_1 u_{m1} & \sigma_2 u_{m2} & \sigma_m u_{mm} & 0 & 0 \end{bmatrix} *$$

$$\begin{bmatrix} v_{11} & v_{21} \cdots & v_{m1} \cdots & v_{n1} \\ v_{12} & v_{22} \cdots & v_{m2} \cdots & v_{n2} \\ v_{13} & v_{23} \cdots & v_{m3} \cdots & v_{n3} \\ \vdots & \vdots & \vdots & \vdots \\ v_{1m} & v_{2m} \cdots & v_{mm} \cdots & v_{nm} \\ v_{1m+1} & v_{2m+1} \cdots & v_{mm+1} \cdots & v_{nm+1} \\ \vdots & \vdots & \vdots & \vdots \\ v_{1n} & v_{2n} & v_{mn} & v_{nn} \end{bmatrix} \quad (3)$$

in a compact notation

$$x_{mn} = \sum_{i=1}^{m} \sigma_i U_{mi} V_{ni}$$

since

$$V_{ni} = V_{in}$$

we obtain

$$X_{mn} = \sum_{i=1}^{m} \sigma_i U_{mi} V_{in}^T \quad \ldots\ldots\ldots\ldots (4)$$

in a general form

$$X = \sum_{i=1}^{r} \sigma_i \, U_i \, V_i^T \quad \ldots\ldots\ldots\ldots\ldots \text{(5)}$$

Where r is the rank of X. If all m rows of X is not a linear combination of the other m−1 rows, we call X has m rows which are linearly independent. In this case the rank of X is equal to m. On the other case, all rows of X is equal in scale so that all rows of X is not linearly independents and the rank of X is equal to one.

From equation (5) $U_i V_i^T$ is a (m x n) matrix whose rank is one which is called the its eigenimage of X. If the rank of X is equal to m, this mean there will be m eigenimages (See Figure 2).

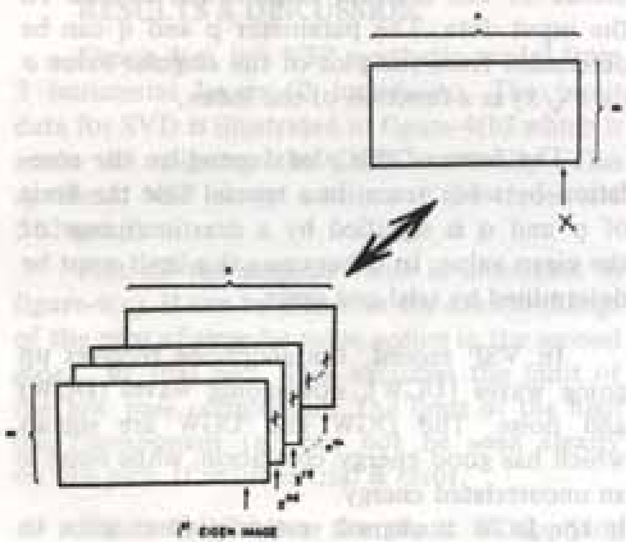

Figure 2. The decomposition/reconstruction of VSP record into/from m eigenimages.

Equation (5) explains that the contribution to reconstruct X from eigenimages is directly proportional to the singular value itself. Since the singular value can always be arranged in order (from the biggest to the smallest), the biggest contributor is contained at the begining of the eigenimage.

By inspection from the matrix equation (1) we can conclude that the (m + 1) th row to n th row from matrix V^T does not contribute to the final result since they are multiplied by the elements of matrix Δ whose values are zero. This fact explains that the size of matrix Δ and V^T can be reduced. The Δ matrix which is originally m x n in size can be reduced to m x m. The V^T matrix whose original size is n x n can be reduced to m x n. In other word, we only need the first m eigenvectors. This properties will certainly reduces the computer memory used to execute the process.

For example for i = 1

$$x(1) = \begin{bmatrix} u_{11} \\ u_{21} \\ u_{m1} \end{bmatrix} [\sigma_1] \, [v_{11} \; v_{21} \cdots v_{m1} \cdots v_{n1}]$$

or

$$x(1) = [\sigma_1] \begin{bmatrix} u_{11} \\ u_{21} \\ u_{m1} \end{bmatrix} [v_{11} \; v_{21} \cdots v_{m1} \cdots v_{n1}]$$

$$= [\sigma_1] \begin{bmatrix} u_{11}v_{11} & u_{11}v_{21} & u_{11}v_{m1} & \cdots & u_{11}v_{n1} \\ u_{21}v_{11} & v_{21}v_{21} & u_{21}v_{m1} & \cdots & u_{21}v_{n1} \\ \cdots & \cdots & \cdots & & \cdots \\ u_{m1}v_{11} & u_{m1}v_{21} & u_{m1}v_{m1} & \cdots & u_{m1}v_{n1} \end{bmatrix}$$

which is the scalar multiplication between the singular value σ_1 and the vector UV^T.

In matrix algebra, the eigenvector of V and U can be computed using the Jacobi method. In the Singular Value Decomposition both vectors can be determined directly by making a strong corelation with respect to U.

The eigen equation is defined by

$$XX^T U = \sigma^2 U \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

or

$$XX^T U = \lambda U \quad \text{.......................} (7)$$

where

$\sigma^2 = \lambda$

multiplying both sides of equation (7) by X^T we obtain

$$X^T XX^T U = X^T U \quad \text{....................} (8)$$

or

$$X^T X (X^T U) = \lambda (X^T U) \quad \text{..............} (9)$$

It can be easily proved that (see Lasmi K, 1990, equation B-6 Appendix B)

$$V = X^T U \quad \text{.........................} (10)$$

Since the size of X^T is (m x m) and U is (m x m) so the size of V is (n x m) or V^T is (m x n).

III. APPLICATION OF SVD TO VSP

In geophysical application, matrix X (m x n) is represented by a seismic record which consists of m traces each has n samples.

If X has m traces which are linearly independent which means the rank of X is full and equal to m. The reconstruction of X required all eigenimages.

For a specific purpose X can be reconstructed using the first p eigenimages. ($1<p<r$). The error in this case can be formulated as

$$\epsilon = \sum_{k=p+1}^{r} \sigma_k$$

where σ to the singular value of X

The eigenimages can be grouped into 3 categories depending on the degree of energy correlation between traces, i.e. the bandpass component, the lowpass component and the highpass component.

The band-pass component (X_{BP}) is obtained by separating signals which has strong correlation (X_{LP}) and signals which has no correlation at all (X_{HP}).

It can be formulated that

$$X_{BP} = \sum_{i=p}^{r} \sigma_i u_i v_i^T$$

$$X_{LP} = \sum_{i=1}^{p-1} \sigma_i u_i v_i$$

$$X_{HP} = \sum_{i=q+1}^{r} \sigma_i \eta_i v_i$$

The choice of p and q depend on the magnitude of the singular value which depend on the input data. The parameter p and q can be determind from the plot of the singular value σ ($\sigma = \sqrt{x}$) as a function of the index.

The form of this plot depend on the corelation between trace. In a special case the limit of p and q is signified by a drastic change of the eigen value. In other cases this limit must be determined by trial and error.

In VSP record, the geophone receives up going waves (UGW), downgoing waves (DGW) and noise. The DGW and UGW are signals which has good energy corelation, while noise is an uncorrelated energy.

If the DGW is aligned vertically from trace to trace this will cause the DGW has better correlation than the UGW. We expect that the DGW is contained in the X_{LP} component, the UGW in the X_{BP} component while the noise in the X_{HP} component.

On the other hand, if the UGW is aligned vertically, the UGW will contained in the X_{LP} and the DGW will be contained in the X_{BP}. The application of SVD to VSP data can be summarized in Fig. 3.

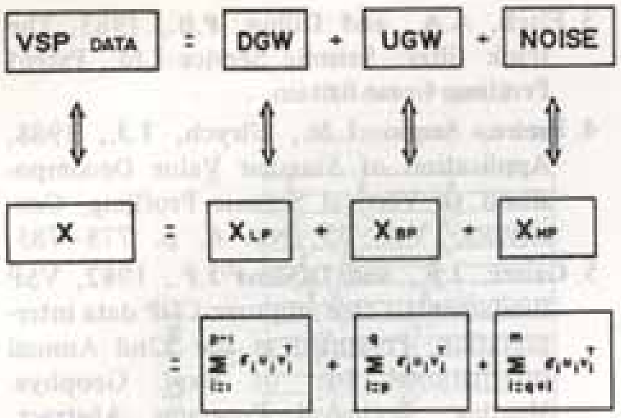

Figure 3. Application of Singular Value Decomposition to VSP data.

IV. EXPERIMENTAL RESULTS & DISCUSSION

Figure-4(a) is a VSP synthetic model from 3 horizontal layers (2 interfaces). The input data for SVD is illustrated in figure-4(b) which is made by aligning the DGW vertically. This synthetic VSP model consists of 24 traces and 512 samples.

Plot of the singular value is illustrated in figure-4(c). It can be seen that the drastic change of the plot of singular value accurs in the second index, so that p=2. This becomes the limit of the low pass component. The limit of the high pass component (q) can not be seen clearly on this plot. It requires trial & error.

Figure-4(d) shows the result of applying the SVD method with p=3 and q=20. The low pass component yield the DGW shown in Fig-4(d). The high pass component yields the noise shown in Figure-4(f). Sum of 20 eigenimages reconstructs the original input data for SVD as shown is Figure-4(g). (cf Fig. 4-b).

To demonstrate the effect of p and q on the final result, a similar experiment was done but by using p=2 and q=23. Figure-4(h) illustrates the low pass component of SVD, Figure-4(i) illustrates the high pass component, while Figure-4(j) illustrates the bandpass component. Figure-4(k) is the DGW after repositioting to the proper arrival time. It can be seen that by changing p and q better output will be obtained.

Experiment using different synthetic VSP model Fig. 5(a) shows different form of the singular value plot Fig. 5(c). The input data is illustrated in Fig. 5(b) which was made from Fig. 5(a) after aligning the first arrival DGW. The application of SVD with p=2 and q=20 yields the low pass component (X_{LP}) illustrated in Figure 5(d) which is the isolated first arrival DGW. The high pass component (X_{HP}) is illustrated in Fig. 5(f). Fig. 5(g) is the UGW which has been shifted in the correct travel time.

The application of SVD to VSP data which has irregular geophone interval is also tested in this experiments. This irregular interval may violate the validity of other approaches such as f–k filter and median filter. The effectiveness of the SVD method for saparating UGW and DGW in the case the geophone interval is irregular is demonstrated in Figure 6.

Figure 6(a) in the input data with irregular geophone interval. Figure 6(b) is similar to Figure 6(a) with the DGW aligned. The plot of the singular value is illustrated in Figure 6(c). Figure g(d) is the lowpass component of SVD using p=2 and q=20. Figure 6(e) is the highpass component, while Figure 6(f) is the band pass component. The UGW after repositioning in time is illustrated in Figure 6(g).

Another experiment is to test the SVD to reject the random noise in VSP record. Figure 7 demonstrates the effectiveness of SVD for this porpose.

Figure 7(a) is the VSP synthetic model with irregular geophone interval and the random noise. Figure 7(b) is the DGW alignment of Figure 7(a) is the input data for the SVD. Figure 7(c) is the singular value of Figure 7(b). By choosing p=2 and q=17 we obtain the lowpass component, the high pass component and or

$$XX^T U = \lambda U \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where

$\sigma^2 = \lambda$

multiplying both sides of equation (7) by X^T we obtain

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On the other hand, if the UGW is aligned vertically, the UGW will contained in the X_{LP} and the DGW will be contained in the X_{BP}. The application of SVD to VSP data can be summarized in Fig. 3.

the band pass component as shown in Figure 7 (a), (e) and (f) respectively. We can see that the random noise has been filtered out from the DGW but it still presents in the UGW.

To reject the random noise from the UGW, the SVD process is repeated and use the last output from the previous process as the input data (Figure 7-f). As before, we need to align the UGW. The result is depicted in Figure 7(g).

The singular value is shown in Figure 7(h). By chosing p=2 and q=17 we get the UGW (low pass component Figure 7-i) and the random noise (the high pass component) as shown in Figure 7(j). Figure 7(k) is the UGW (Figure 7-l) after the arrival time has been reshifted to their proper time. We see that the UGW component is now free from the random noise.

V. CONCLUSION

1. The Singular Value Decomposition (SVD) has been tested to be a powerful tool for separating upgoing and downgoing wave in VSP record.
2. The cat off values (p and q) can be determined from the drastic change in the singular value plot or by the and error, especially for determining q.
3. The random noise can be automatically suppressed.
4. The SVD can be easily used to process the VSP data with irregular geophone interval.

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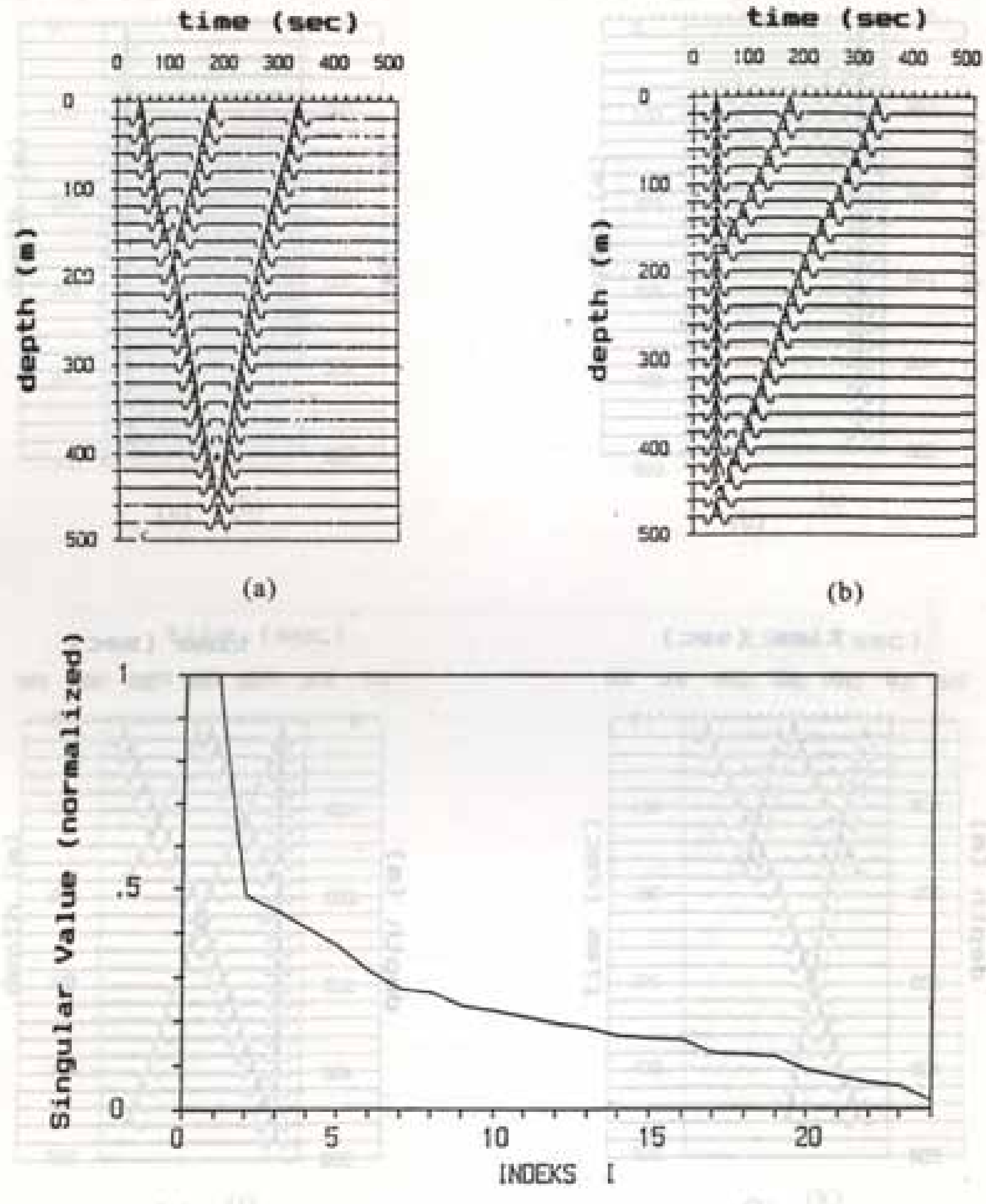


Figure 4. (a) Synthetic VSP data from three horizontal layers model (two interfaces).
(b) The DGW alignment of Figure 3(a).
(c) The Singular value plot of Figure 3(b).

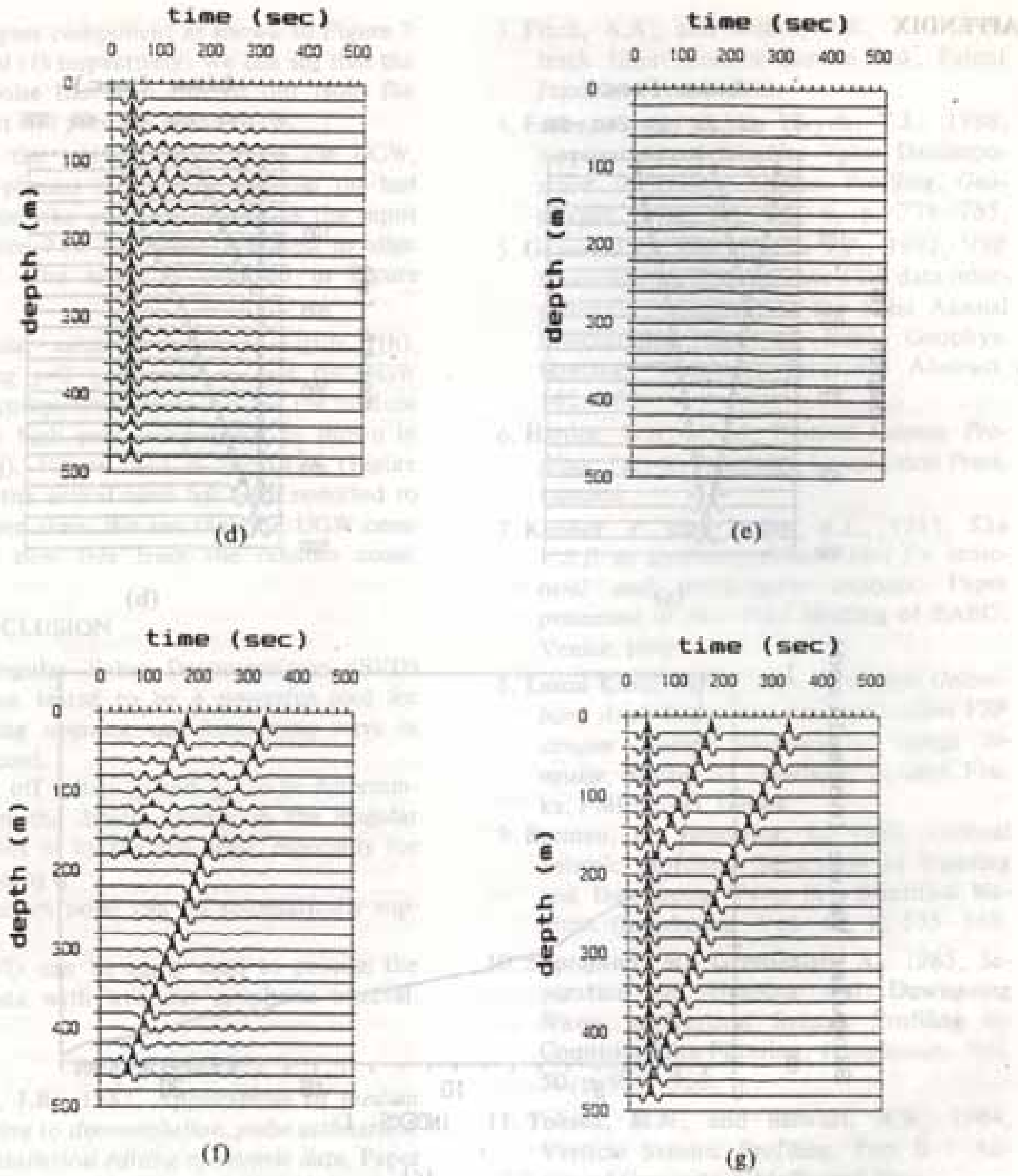
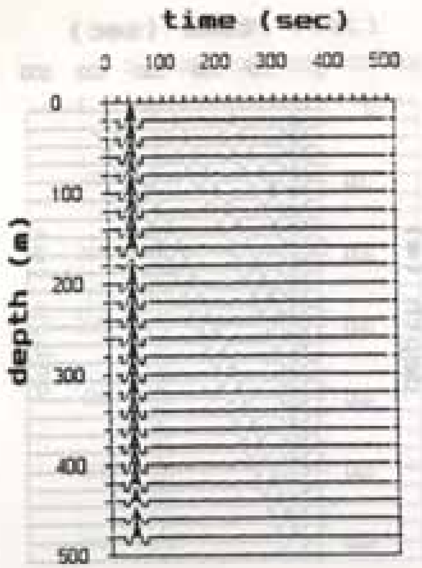
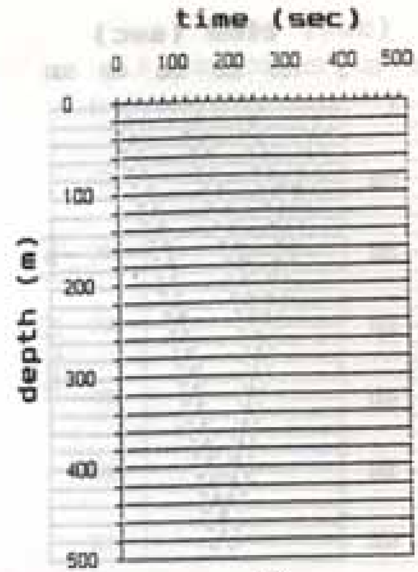


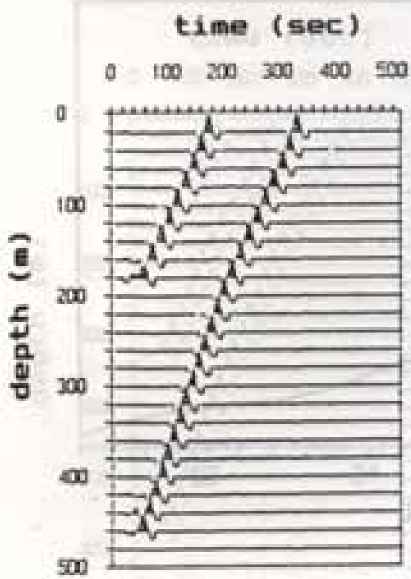
Figure 4. (d) The lowpass component of SVD using p = 3.
(e) The highpass component of SVD using q = 20.
(f) The bandpass component of SVD using p = 3 and q = 20.
(g) Sum of the first 20 eigenimages.



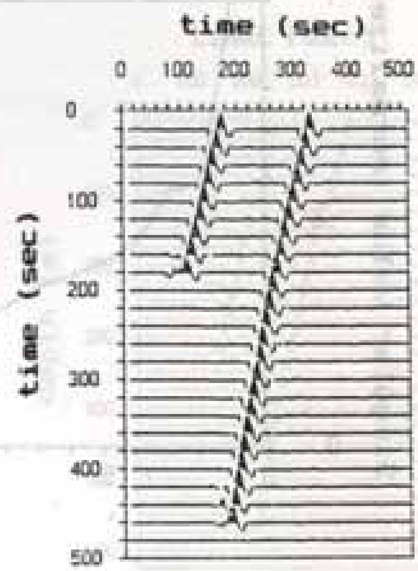
(h)



(i)

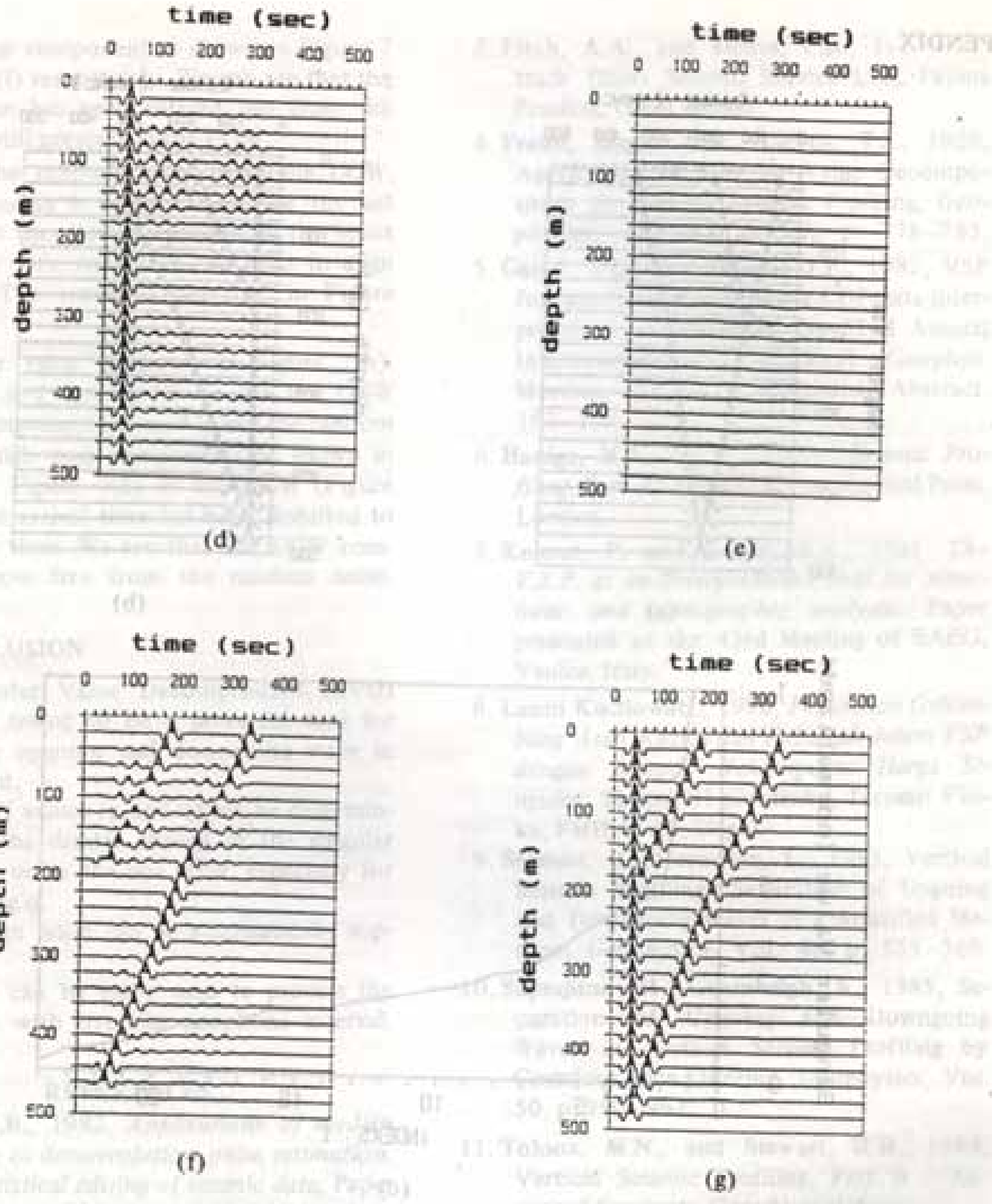


(j)



(k)

Figure 4. (h) Same as Figure 3(d) but p = 2
(i) Same as Figure 3(e) but q = 23
(j) Same as Figure 3(f) but p = 2 and q = 23.



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(e) The highpass component of SVD using q = 20.
(f) The bandpass component of SVD using p = 3 and q = 20.
(g) Sum of the first 20 eigenimages.

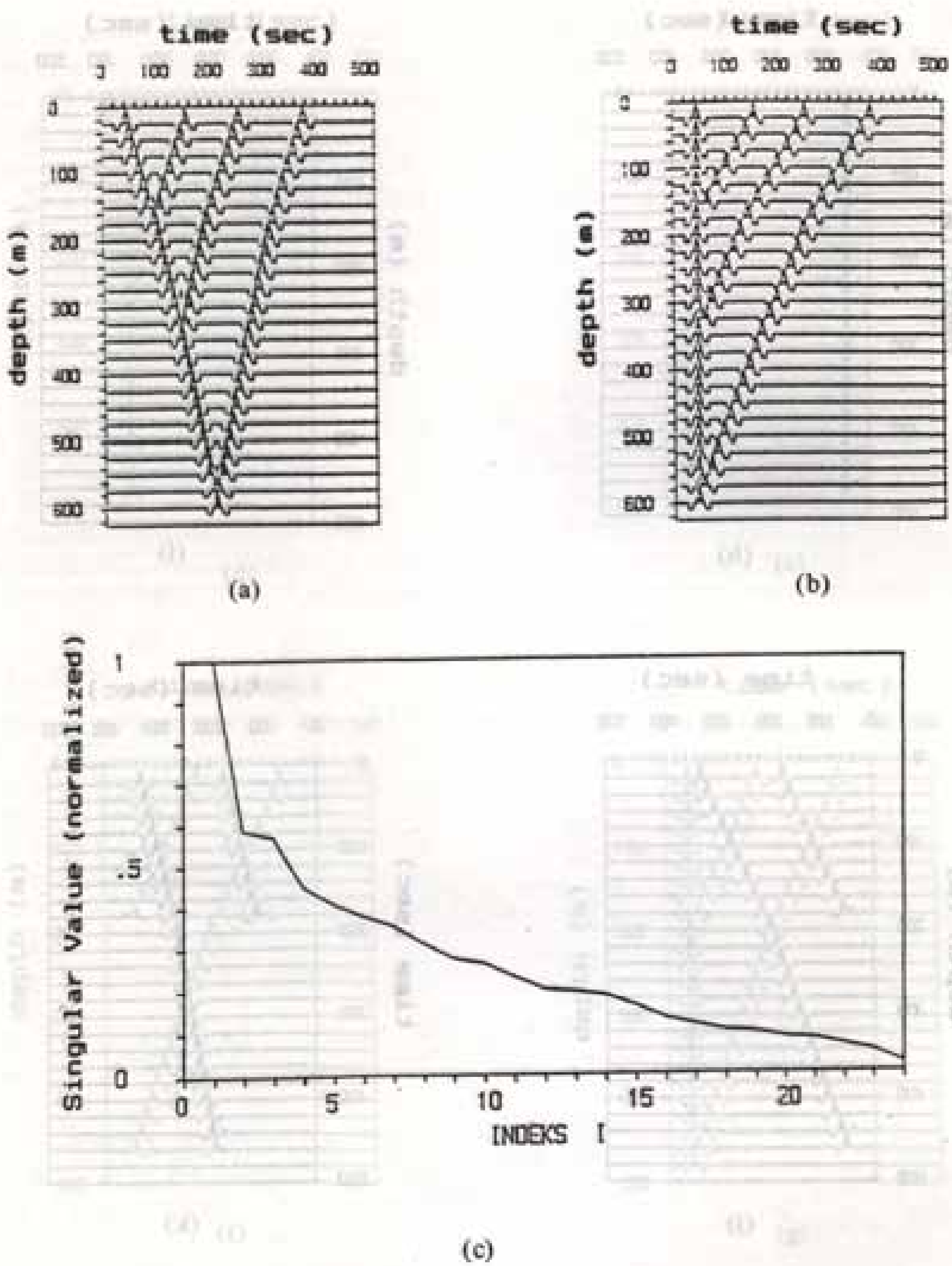

Figure 5. (a) Synthetic VSP data from four horizontal layers model (three interface)
(b) The DGW alignment of Figure 4(a)
(c) The Singular value plot of Figure 4(b).

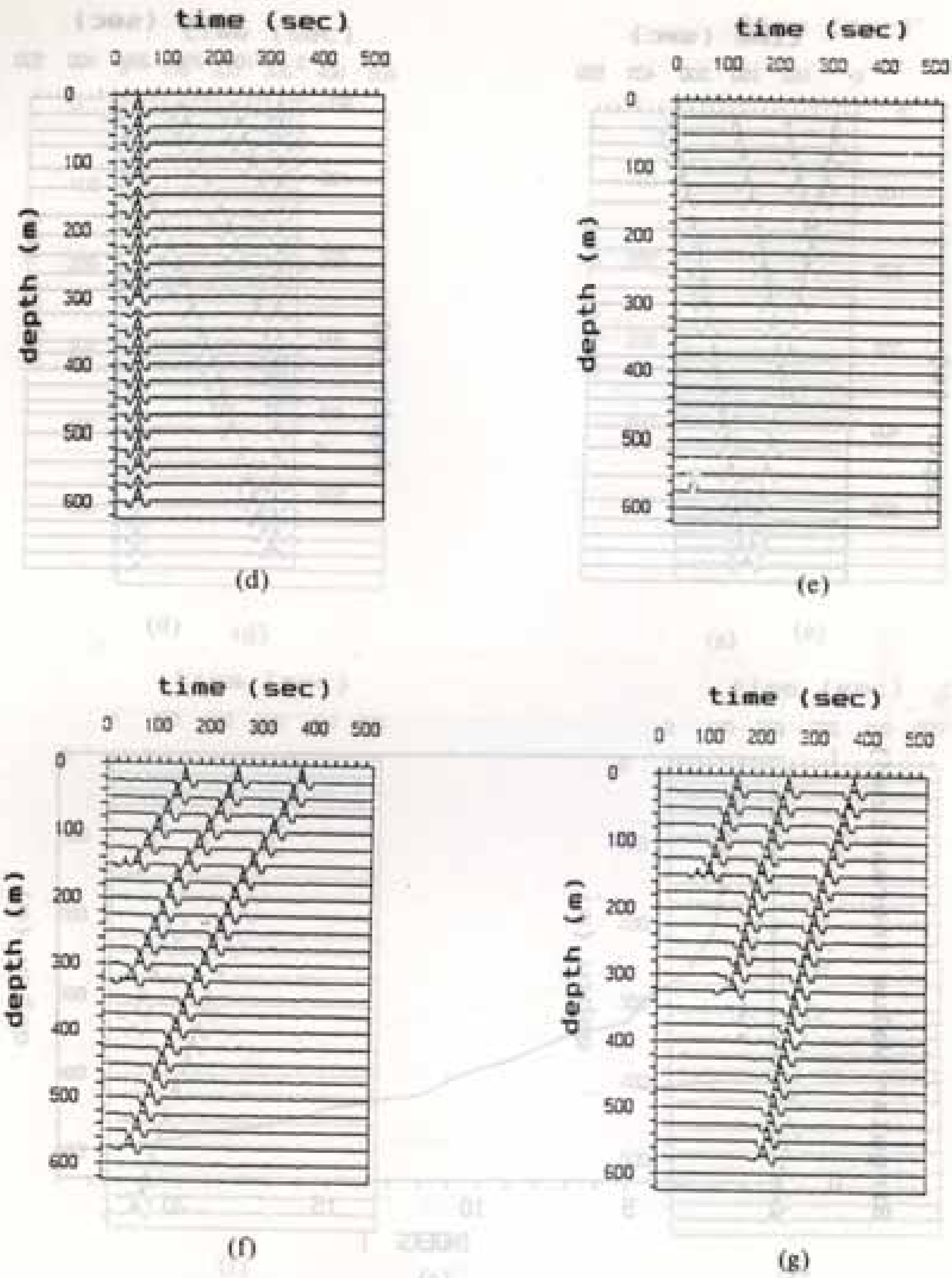

Figure 5. (d) The lowpass component of SVD using p = 2.
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(f) The bandpass component of SVD using p = 2 and q = 20.
(g) The UGW after reshifting to their proper travel time.

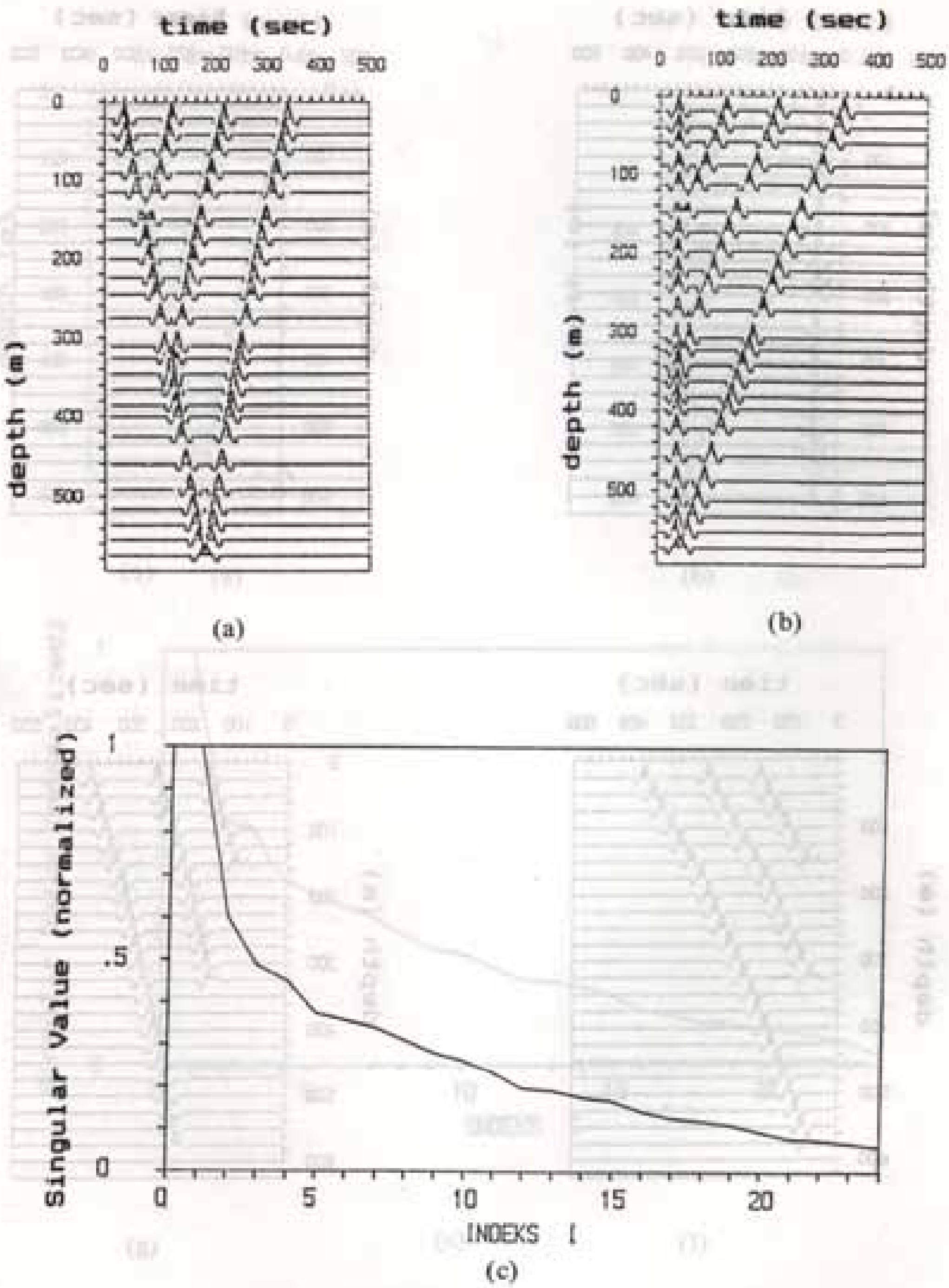
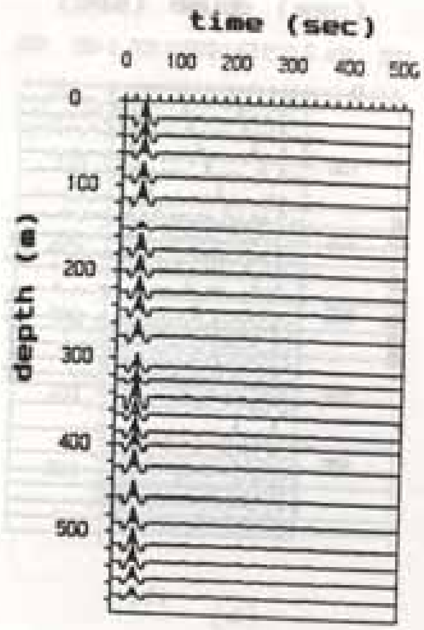
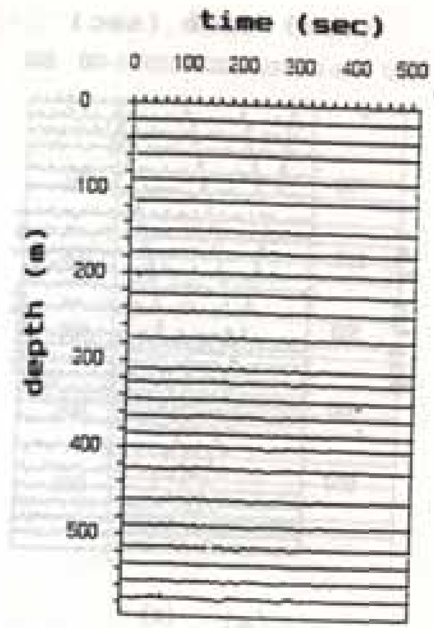


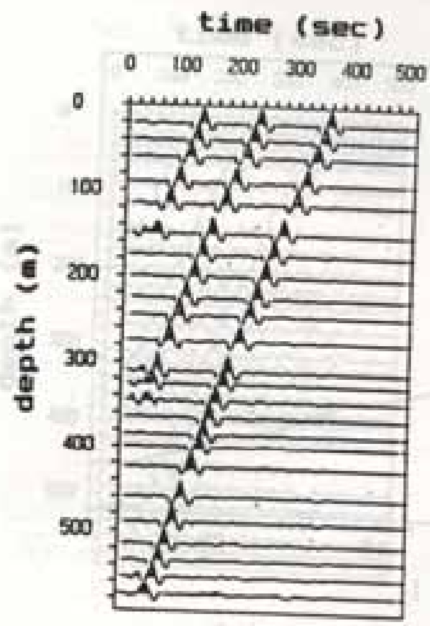
Figure 6. (a) Synthetic VSP data with irregular geophone interval.
(b) The DGW alignment of Figure 5(a).
(c) The Singular value plot of Figure 5(b).



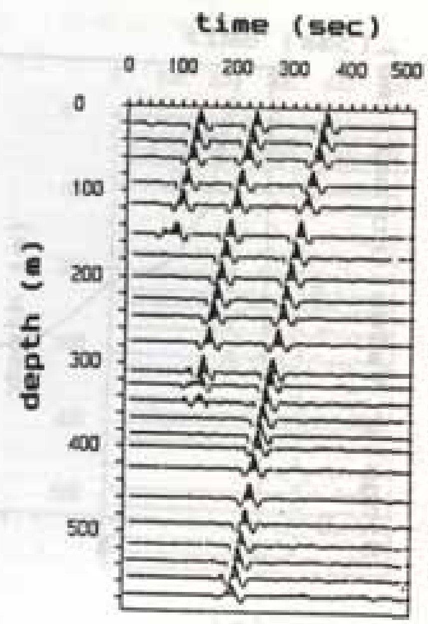
(d)



(e)

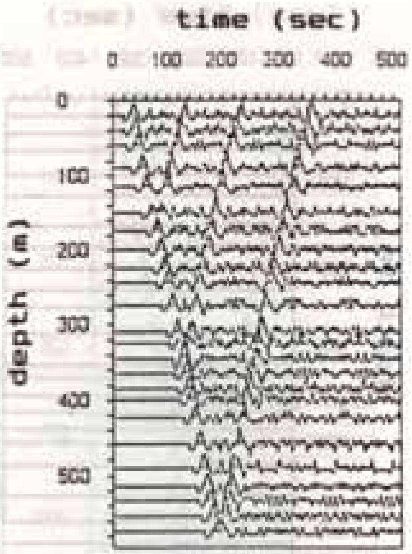


(f)

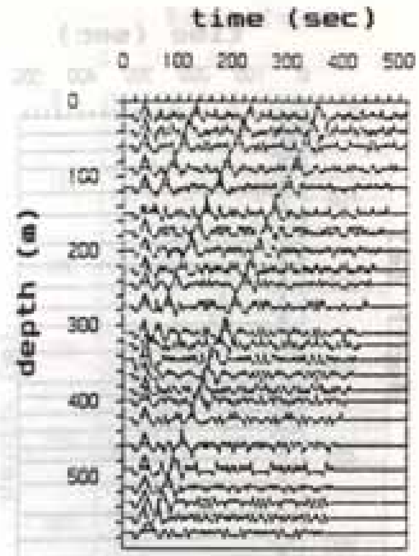


(g)

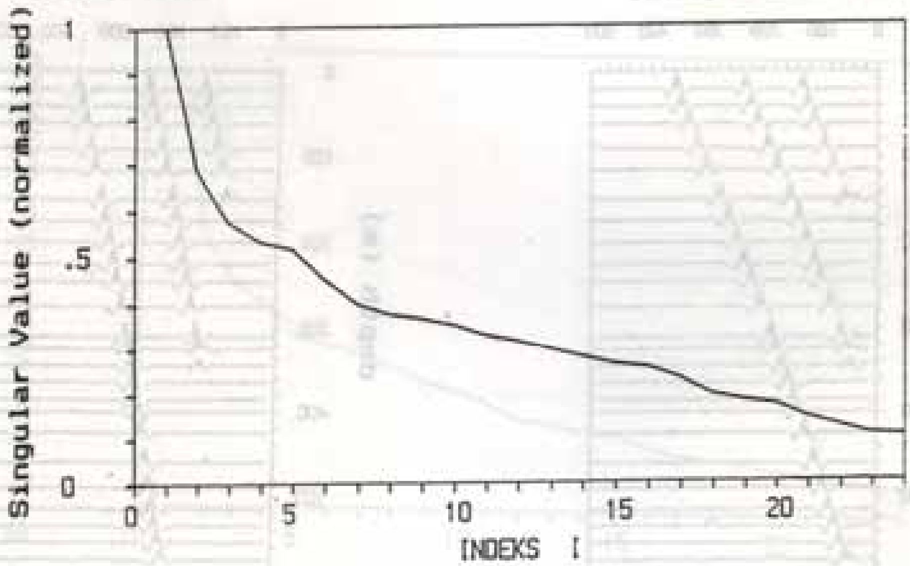
Figure 6. (d) The lowpass component of SVD using p = 2
(e) The highpass component of SVD using q = 20.
(f) The bandpass component of SVD using p = 3 and q = 20.
(g) The UGW after reshifting to their proper travel time.



(a)



(b)



(c)

Figure 7. (a) Synthetic VSP data with random noise and irregular geophone interval.
(b) The DGW alignment of Figure 6(a)
(c) The Singular value plot of Figure 6(b).

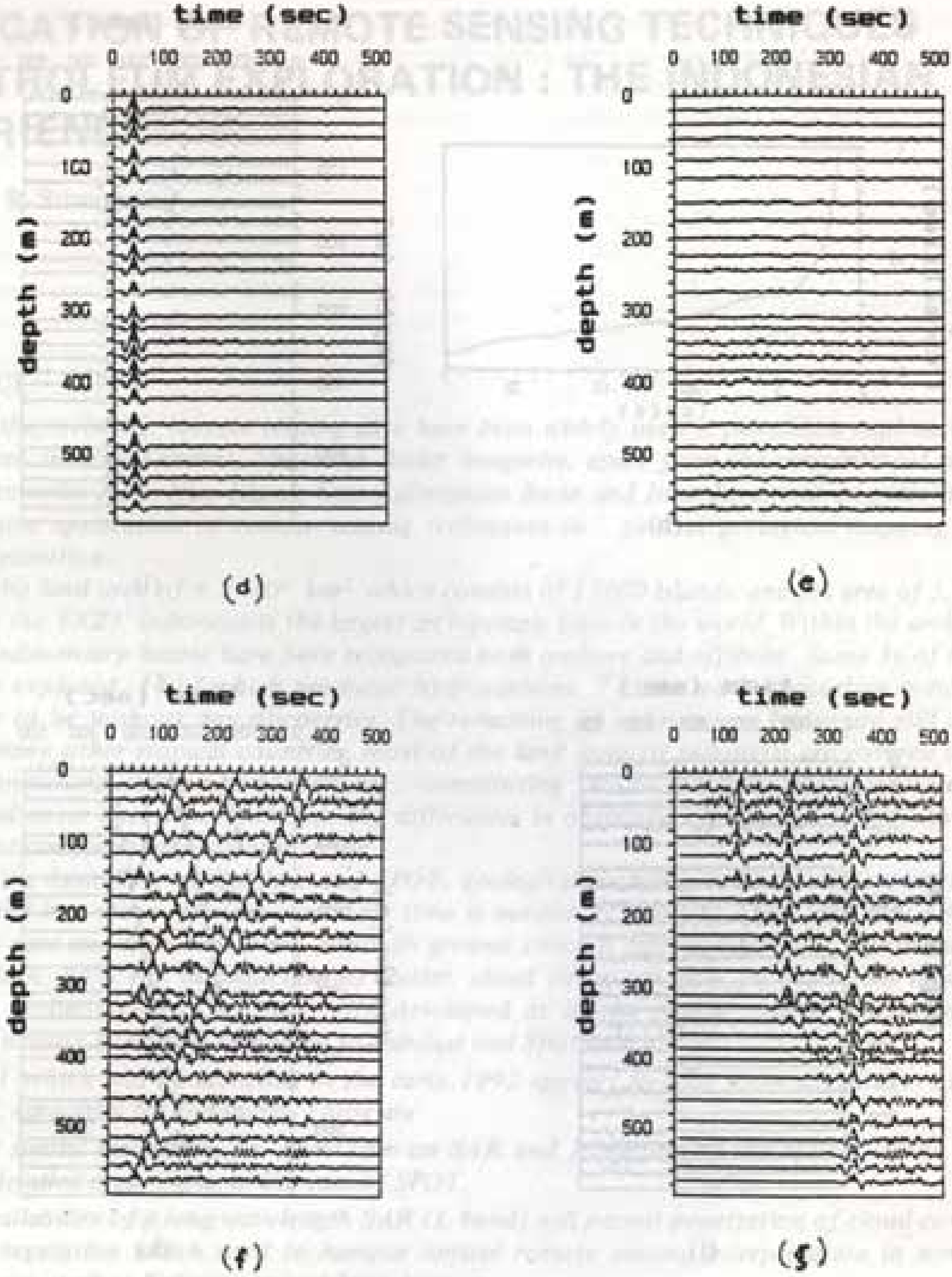


Figure 7. (d) **The lowpass component of SVD using p = 2**
(e) **The highpass component of SVD using q = 17**
(f) **The bandpass component of SVD using p = 2 and q = 17.**
(g). **The UGW alignment of Figure 6(f).**

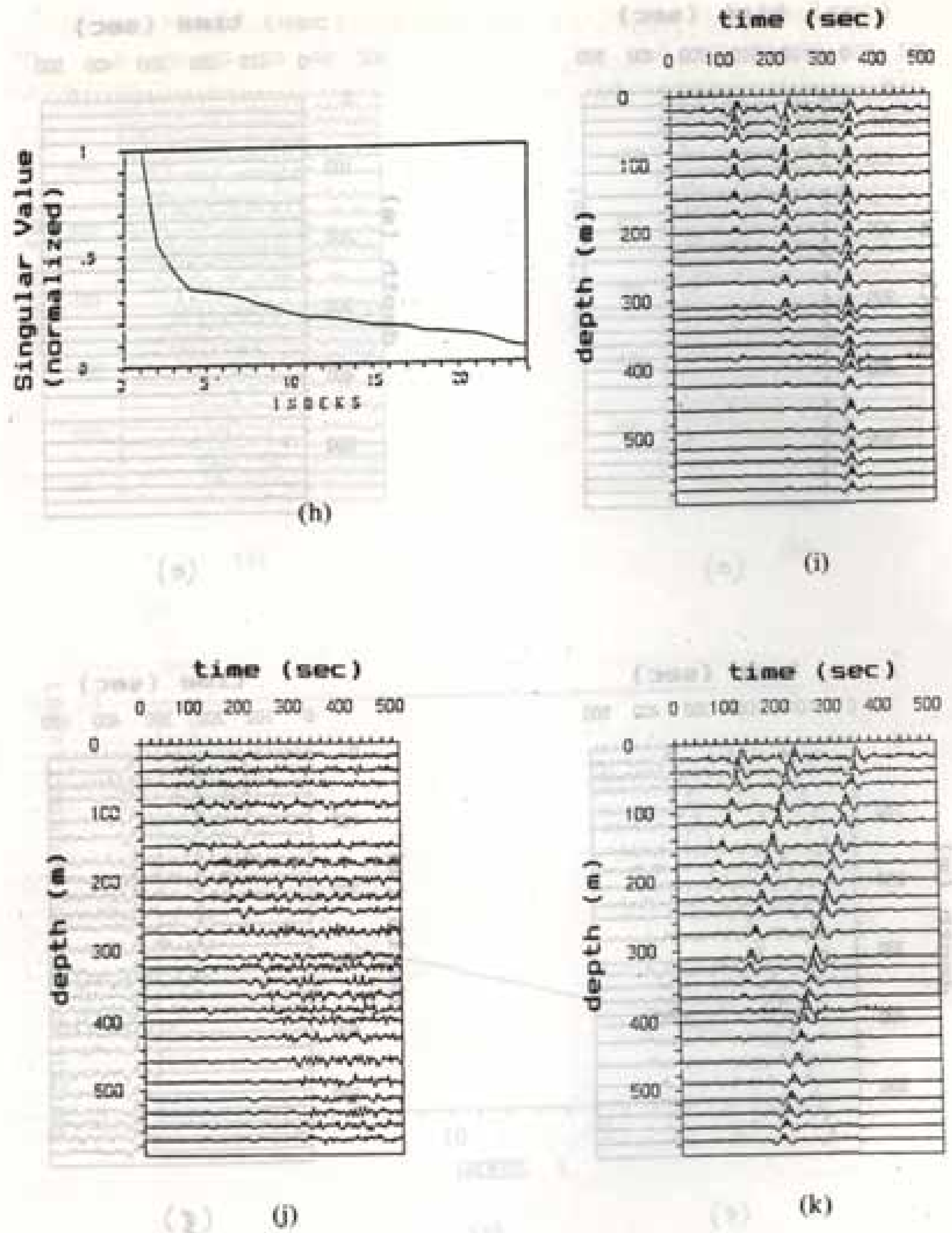


Figure 7. (h) The Singular value plot of Figure 6(g)
(i) The lowpass component of SVD using p = 3.
(j) The highpass component of SVD.
(k) The UGW after static reshifting.