TWO-DIMENSIONAL INTERPOLATION OF POTENTIAL GEOPHYSICS DATA

by
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ABSTRACT

Many types of geophysical survey involve making numerical observations at a large number of localities in a so-called survey area. In order to display the results of such a survey in a way which can be easily assimilated by the interpreter, they must be plotted in a map form, with the X and Y values (geographical position) associated with each data point. In most cases, contouring of these plotted values is employed to further enhance their appearance and usability. Commonly, particularly where computers are employed, there are two stages in this process, namely the two-dimensional interpolation of the field observations onto the nodes of a regular grid (the so-called gridding interpolation), then contouring the gridded data.

The development of popular micro-computers which are now widely used, such as IBM PC or its compatibles, has become a great convenience in the processing of potential geophysics data, such as gravity or magnetic data. The automatic contouring process, also the gridding interpolation, which were mainly done on the main frame, now can be done on the micros. A computer program has been developed for this purpose, i.e. the gridding processes, runs on an IBM PC micro-computer.

I. INTRODUCTION

Contour maps have been used by geophysicists for many years as one way of presenting a variety of data sets. In producing such a contour map, manual interpolation was carried out among data points to determine the position where a contour line will pass, and then the contours were drawn through the positions of equal value interpolated (e.g. the isogal or the isogamma).

During recent years, contouring by machine has become increasingly common and a variety of computer programs has been available for contouring any data. In practice, this involves two essential stages, namely :

1. the two-dimensional interpolation of the scattered field observation onto nodes of a regular grid, the so-called **gridding interpolation process**,
2. threading the contour lines among the gridded data, the so-called **contouring process**.

The development and the advent of micro-computers, have become a great convenience in those processes, and also a great help in interpreting geophysical data (Reeves and McLeod, 1983).

II. THE GRIDDING INTERPOLATION

A. *The need for gridding interpolation*

A common feature of almost any contouring program is that it needs gridded data. For convenience, this is generally already stored in a data file. There are therefore two essential stages in the contouring process, that is :

- the first stage is to interpolate the data from the actual profiles or from the data points, to produce the gridded data set.
- contouring is then performed from the gridded data as the second stage of the process.

In Fig. 1, the role of interpolation with respect to data processing and contouring has been described. It implies that if either data processing or contouring is to be done, interpolation has to be carried out in advance. In the case where old data are only available in map form, digitizing along (imaginary) profile lines has to be carried out first. One of the more important advantages is that the gridded data can be stored into a file ready for many types of data processing, since almost all methods for 2-D data processing are only available for equispaced data, i.e. the gridded data (Reford and Sumner, 1964, Fuller, 1967, Nettleton, 1976).

The next problem is to find a satisfactory method of gridding interpolation which will yield a good result. Good interpolation means that the desirable features will be :

a. the interpolated values cannot be drastically different from the values of nearby data points; points of known value must be honoured.
b. no artificial anomalies are created in areas having no data control points.
c. pleasing to the eye, easy to read, well labelled.
d. no excessive use of computer time.

B. The gridding method developed

The problem of two-dimensional interpolation can be outlined in a simple way. Let the value of z_i of an observed variable be known at a fixed number of locations (x_i,y_i) in an area. The objective is to determine approximate values of the variable at points other than the known locations. The particular problem is concerned with doing this interpolation for grid points located at the intersections of a rectangular grid. Any method, in order to be suitable for treatment of large volumes of data, should be conveniently adaptable to computer use.

There are several methods that can be used to grid the data; however, a method which was used and developed is discussed in this paper, the so-called **finite difference method.** This method is based on the assumption that the desired surface which passes through all data points obeys some form of differential equation. The equation is then approximated by finite differences and solved iteratively (Briggs, 1974; Swain, 1976).

A finite difference method is an approximate method of solving differential equations numerically, by replacing derivatives at a point by difference quotients over a small interval. An imaginary surface **u** (like a thin elastic sheet) is bent to fit the observation points exactly. The two-dimensional differential equation which describes such a

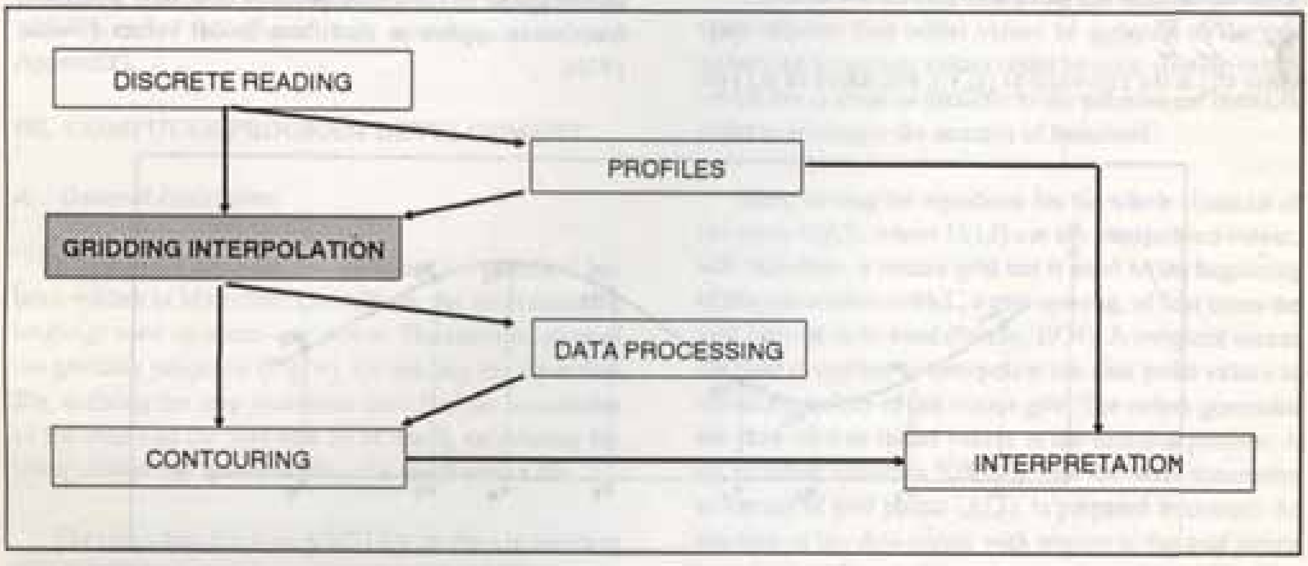

Figure 1. The role of interpolation with respect to data processing and contouring.

surface is the biharmonic equation (Briggs, 1974) :

$$\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2\,\partial y^2} + \frac{\partial^4 u}{\partial x^4} = 0 \qquad [1]$$

for (x,y) not coinciding with one of the points (x_k, y_k) with boundary conditions :

$$u(x_k, y_k) = u_k \;;$$

$$\frac{\partial^2 u}{\partial x^2} = 0\;;\;\; \frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2}\right) = 0 \qquad [2]$$

in one-dimensional form this function describes a spline (Fig. 2).

It can be shown that of all functions u that conform to the given set of boundary conditions (i.e. data points), the one that obeys this equation has the least total squared curvature, i.e.

$$C(u) = \iint \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) dx\,dy \qquad [3]$$

is minimum (Briggs, 1974). That is, u is the smoothest surface passing through the data points, and is in fact equivalent to fitting a two-dimensional spline to the data.

For a set of grid points values $u_{i,j}$ at location (x_i, y_j), where $i=1,2,\ldots,I$ and $j=1,2,\ldots,J$, Eq. [3] can be written in the discrete form, putting x and y equal to unity :

$$C = \sum \sum (C_{i,j})^2 \qquad [4]$$

where $C_{i,j}$ is the curvature at (x_i, y_j), a function of $u_{i,j}$ and some neighbouring grid values. The simplest approximation to the curvature at (x_i, y_j) is

$$C_{i,j} = u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i+1,j+1} - 4\,u_{i,j} \qquad [5]$$

To minimize the sum of C, the function is differentiated with respect to $u_{i,j}$, and set to equal zero and this produces is the biharmonic equation for a grid point at (i,j). The lay-out of the grid points is shown in Fig. 3.

The biharmonic equation is :

$$S_1 + 2\,S_2 - 8\,S_3 + 20\,u_{i,j} = 0 \qquad [6]$$

where :

$$S_1 = u_1 + u_5 + u_9 + u_{13}$$

$$S_2 = u_2 + u_4 + u_{10} + u_{12}$$

$$S_3 = u_3 + u_6 + u_8 + u_{11}$$

and the equation is solved iteratively, using the Gauss-Seidel method (Kreyszig, 1979) :

$$u_{i,j}^{\,p+1} = \frac{\left[\,8S_3^{\,p} - S_1^{\,p} - 2S_2^{\,p}\,\right]}{20} \qquad [7]$$

where the index p indicates the p^{th} iteration.

A practical problem with implementing this method is that the iterative method which is used to solve the equations requires initial values to be assigned to the grid points. In order to minimize the number of iterations and also to speed up the convergence, a quadratic weighting function is applied to find these initial values (Swain, 1976).

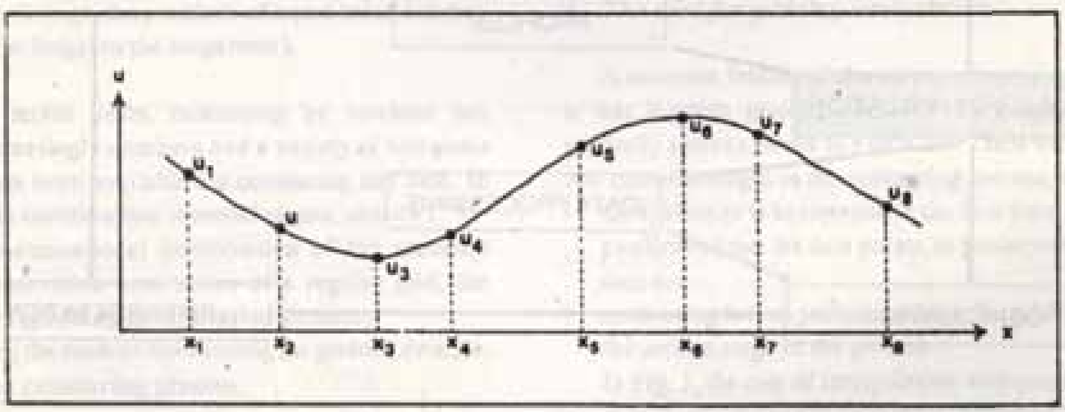

Figure 2. Biharmonic equation in one-dimension, $\partial^4 u/\partial x^4 = 0$, with boundary conditions at $x_1, x_2, x_3, \ldots x_8$ describes a spline

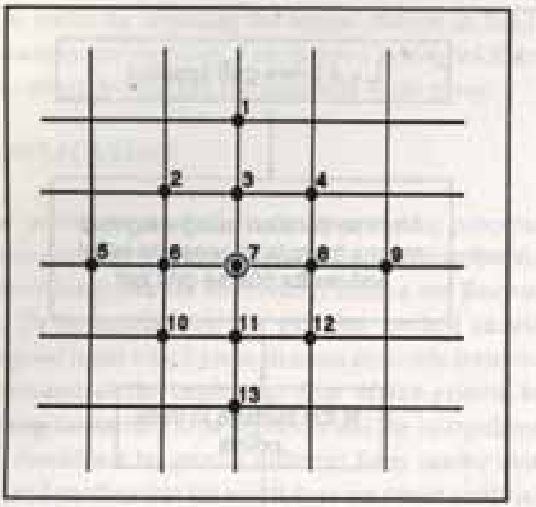

Figure 3. Grid points used by two-dimensional biharmonic difference equation.

The choosing of the grid interval is taken with the consideration that the actual data points will coincide with the grid points. However, since they are irregularly distributed in the area, most of them do not coincide. If they do, the above Eq. [6], which is called the *grid point equation*, is used. To honour the actual data, in the general case that a data point does not coincide, another equation, which is called the *off-grid point equation*, is used (see Appendix).

III. COMPUTER PROGRAM DEVELOPMENT

A. *General description*

A computer program for gridding interpolation has been written in MicroSoft QuickBasic, the most common language used on micro-computers. The essential steps of the gridding programs (Fig. 4), are reading the input data file, defining the map parameter used (i.e. the boundaries of the map and the grid size to be used), calculating the interpolation and finally storing the result onto a file.

The input data file is an ASCII file, written in standard GEOSOFT format (Reeves and McLeod, 1986), and the program will read 3 columns from the file, which will be treated as X- , Y- and Z-values. While reading the input data file, the program will find the minimum and maximum of X- and Y-values. If the user does not agree with these values, he may use his own, and an option is offered for this purpose. Another option follows, for using the grid size, whether to use a new number or not. Before the interpolation begins, the user is asked to enter a file name, onto which the result will be stored.

The output file is a binary file, for the sake ot disk space, headed by the number of rows and columns, X-min and X-max values, Y-min and Y- max values, Z-min and Z-max values, followed by the gridded data.

B. *Gridding interpolation program*

The program developed is based on a program which was written, in FORTRAN language, by Swain (1976). Some modifications and improvements are made to be adapted in micro-computer, which is written in Microsoft QuickBasic. The flow-chart of the main part of this program, i.e. the way of calculating the grid point values, is shown in Fig. 5.

The iterative method in solving the biharmonic equations requires that initial values be assigned to the grid points. Although any values could be used, starting values which are as close as possible to the solution are better, in order to minimize the number of iterations.

Since solving the equations for the whole elements of the array U(I,J), where U(I,J) are the interpolated values, will take time, a coarse grid net is used at the beginning of the calculation, with L, a grid spacing, of four times the grid interval to be used (Swain, 1976). A weighted means formula is applied to interpolate the data point values to obtain the values of this coarse grid. The values generated are then used as initial values in the iteration process. A set of string variables IU$(I,J), with the same dimension as the set of grid points U(I,J), is prepared to control the position of the data points with respect to the grid points evaluated in fitting the surface to data points. If a data

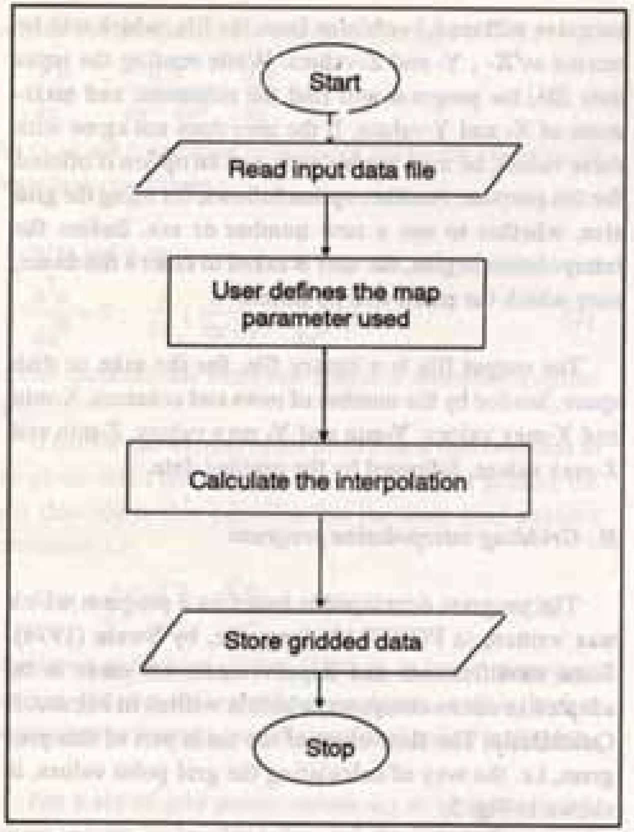

Figure 4. The essential steps of the interpolation program.

point lies nearby a grid point (I,J), a flag is assigned to IU$(I,J), and the *off-grid point* equation will be used for this grid point. Otherwise, the *grid point* equation will be used.

After an iterative solution is determined, the coarse grid spacing is halved, i.e. L is reduced to L/2, and the grid values at (I,J) are copied to the new grid points (I+L,J), (I,J+L) and (I+L,J+L). This new grid net is again fitted to the data points, and a new flag is assigned to the appropriate elements of the IU$(I,J), according to the position of the data points to the new grid net. Keeping the previous values at (I,J) fixed by setting the appropriate elements of the string IU$(I,J) with a flag of -1, the iteration is repeated to obtain the new values at (I+l,J), (I,J+L) and (I+L,J+L).

The procedure is repeated, namely : halving of the grid

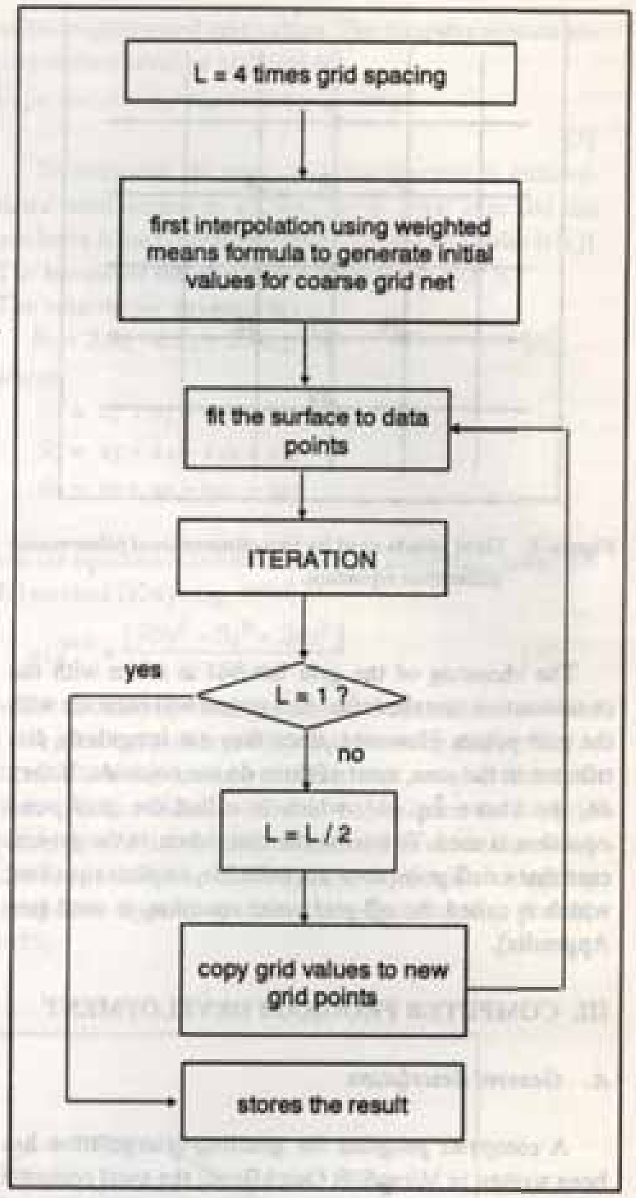

Figure 5. Flow-chart of the main part of the program, showing the way of calculating the grid point values.

spacing, copying the grid values to the new grid points, fitting the new grid net to the data points and solving the equations, until the required grid size is obtained (Fig. 6). The iteration process is only applied to the new gaid points by keeping the previous grid values fixed, and conver-

gence is tested by retaining the largest change in U(I,J) that occurs in one iteration. The iteration is stopped if the largest change is less than the threshold value given.

IV. APPLICATION

The problems of satisfactorily interpolating potential field data onto a grid system from the original distribution of observation points are not trivial (Paterson and Reeves, 1985). To be satisfactory, the gridding method should yield a good result which presents some desirable features, as mentioned at the beginning. One of the criteria in examining the method to be applied is that the interpolated values should not be greatly different from nearby data points, and another that the result does not create artificial anomalies.

As an example, the method was applied to gravity data for an area covering approximately 28 by 21 km, and 255 gravity stations were established along accessible roads and tracks with an average station spacing of 0.5-1 km. The data were gridded using a grid interval of 0.75 km and 0.5 km, i.e. 33 by 29 and 49 by 43 respectively.

The program was run on an 80386 machine with a math co-processor 80387, using a threshold value of 0.1 for stopping the iteration processes; it all took only about 2 minutes. The results of the gridding interpolation were then contoured, as shown in Fig.7 and Fig.8.

V. CONCLUSION

A method of two-dimensional interpolation has been discussed, i.e. the finite difference method, and developed to be used on micro-computer.This method provides a good result in interpolating the common type of gravity or magnetic data, which are irregularly distributed in the survey area.

The advantage of having gridded data that are already stored in a file is that they cannot only be used for automatic contouring, but many other purposes, such as regional and residual anomaly separation, two-dimensional filtering, or any other 2-dimensional data processing.

REFERENCES

Briggs, I.C., 1974. "Machine contouring using minimum curvature", *Geophysics*, **39**, 1, pp.39-48

Fuller, B.D., 1967. "Two dimensional frequency analysis and design of grid operators", *in Mining Geophysics*, Vol II, S.E.G., Tulsa, pp. 658-708

Kreyszig, E., 1979, *Advance Engineering Mathematics*, John Wiley & Sons, Inc., New York, 939 pp.

Nettleton, L.L., 1976, *Gravity and Magnetics in Oil Prospecting*, McGraw Hill Book Co., New York, 464 pp.

Paterson, N.R. and Reeves, C.V., 1985, "Application of gravity and magnetic surveys: The state-of-the-art in 1985". *Geophysics*, **50**, 12, pp.2258- 2594

Reeves, C.V. and McLeod, I.N., 1983, "*Modelling of potential field anomalies - some application for the micro-computer*", *First Break*, 1, 8, pp.18-24

Reeves, C.V. and McLeod, I. N., 1986, *A standard structure for geophysical data file on popular micro-computer*", *First Break*.

Reford, M.S. and Sumner, J. S., 1964, "Review of aeromagnetic". *Geophysics*, **29**, pp.482-516

Swain, C. J., 1976, "A Fortran IV program for interpolating irregularly spaced data using the difference equations for minimum curvature", *Computers and Geoscience*, 1, pp.231-240

APPENDIX

THE OFF-GRID POINT EQUATION

If $C_{i,j} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ at point (x_i, y_i) [A-1]

is substituted into Eq. [1], it gives the difference equation:

$$C_{i,j-1}+C_{i,j+1}+C_{i-1,j}+C_{i+1,j}-4\,C_{i,j}=0 \qquad \text{[A-2]}$$

The use of the difference equation [5] in [A-2], results in the common difference equation [6]. For the off-grid points, however, an expression for $C_{i,j}$ is needed, which uses values of u at discrete points not lying on a regular grid.

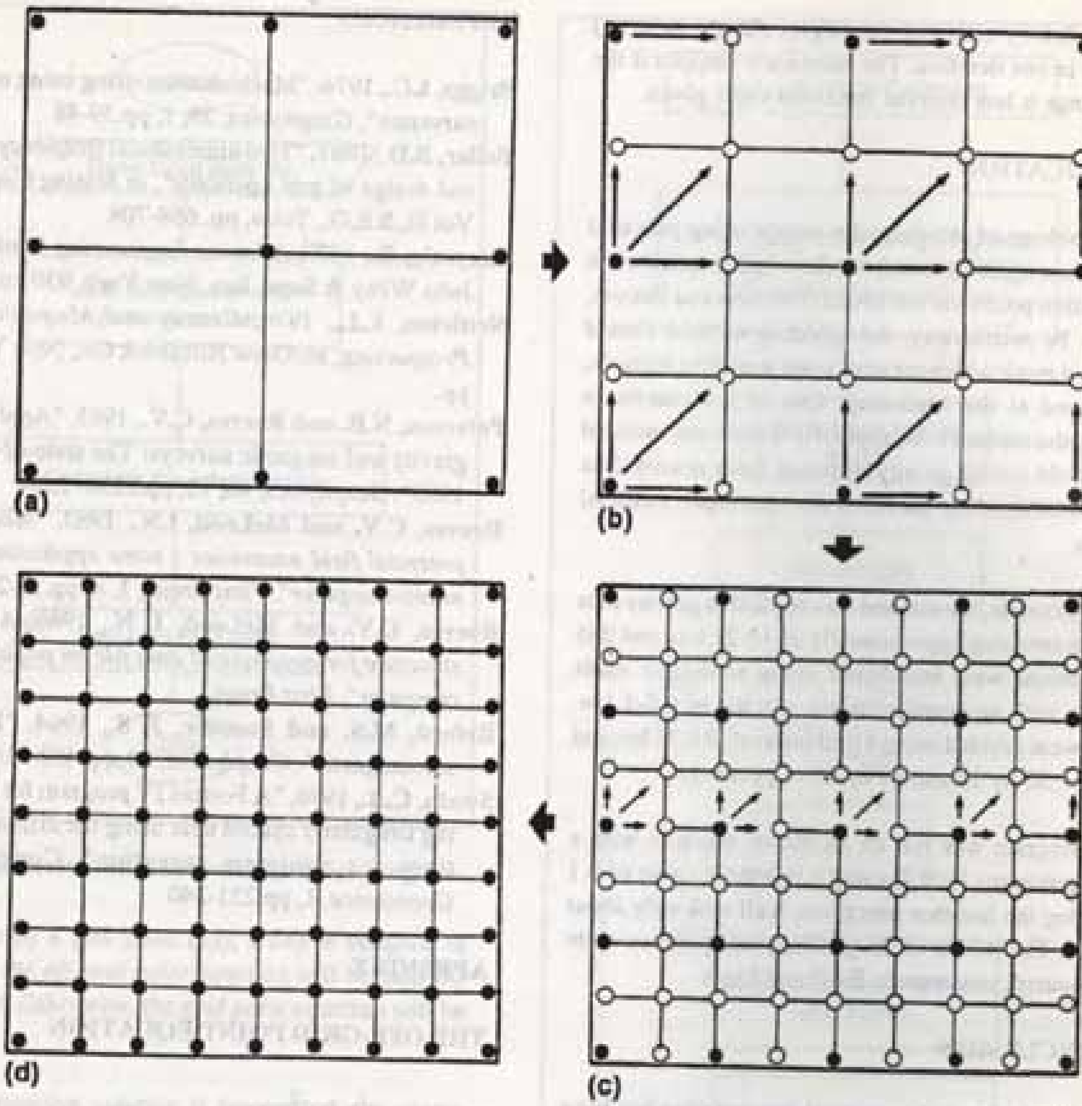

Figure 6. Steps in calculating the grid point values in the finite difference method.

a. a coarse grid net is generated and an iterative solution is determined.

b. the grid spacing is reduced by a factor of two, the previous grid values are assigned to the new grid points and obtain the iterative solution for this grid net.

c. repeat the step (b) until the required grid size is reached.

d. final gridded data

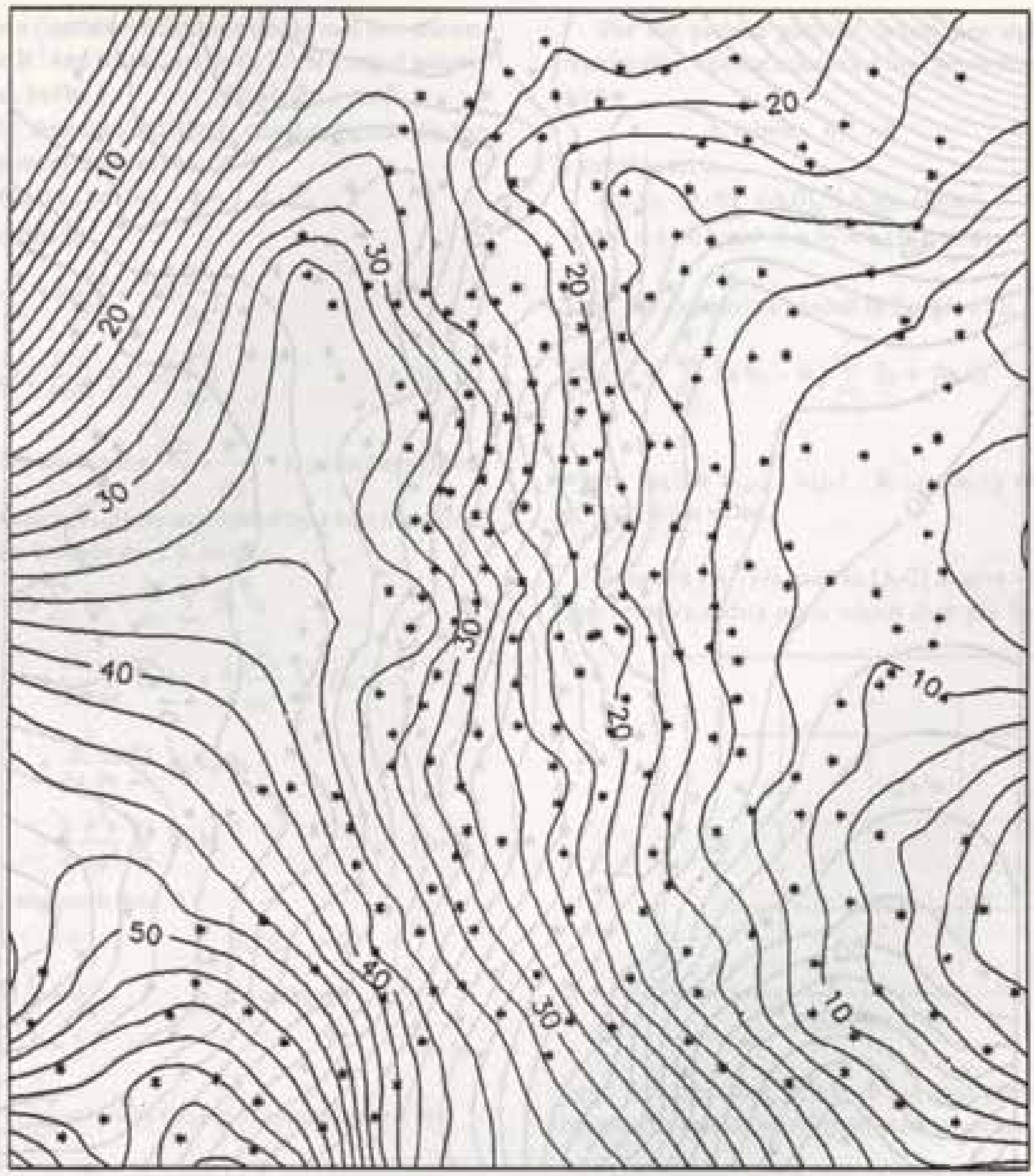

Figure 7. Gravity anomaly map, contoured based on a 33 by 29 grid interpolation result. Gravity stations are shown with stars; note that as the western and northwestern parts of the area have no gravity stations, there is no judgment or control on those interpolation values.

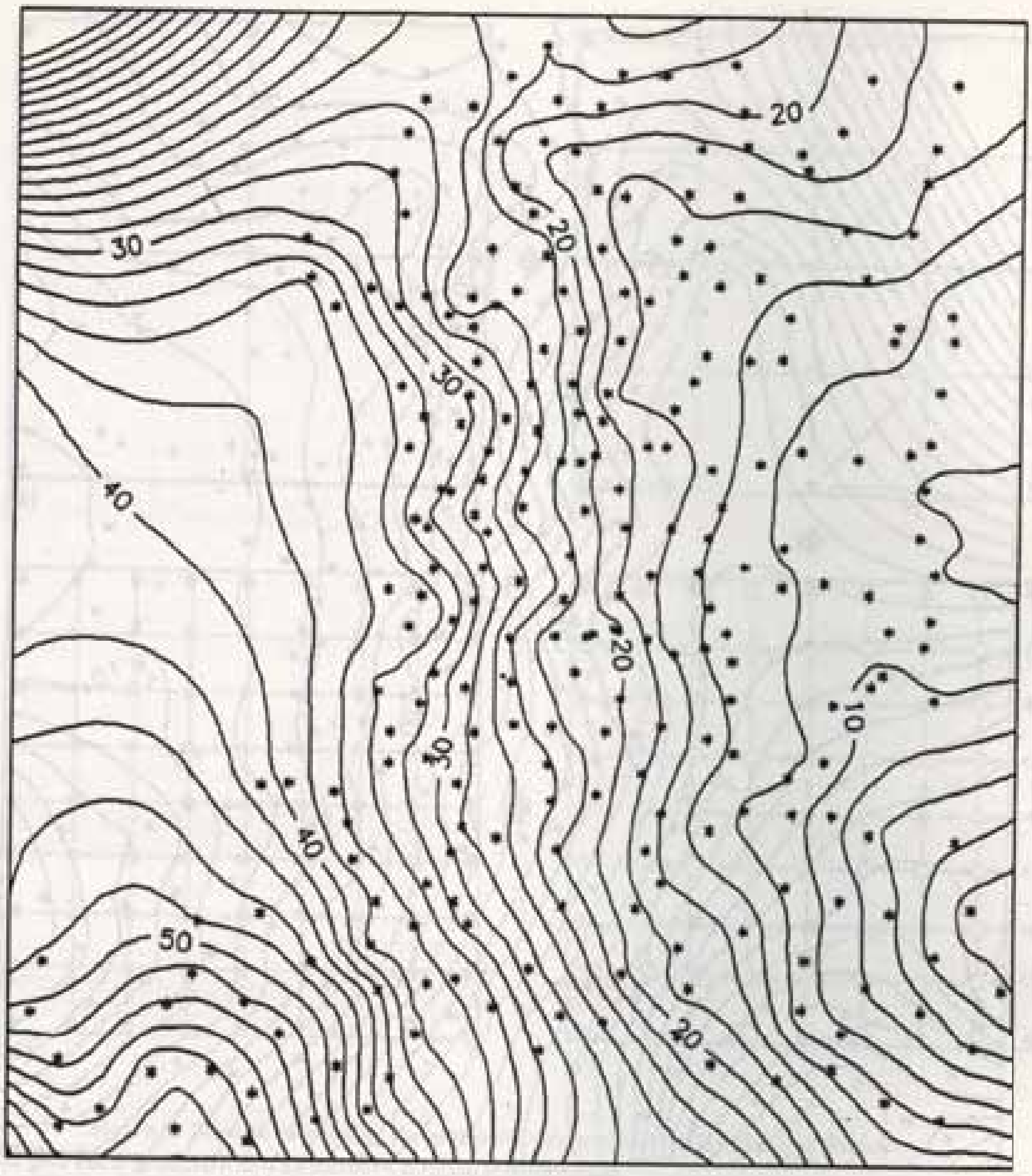

Figure 8. Gravity anomaly map, contoured based on a 49 by 43 grid interpolation result. Gravity stations are shown with stars; note that as the western and northwestern parst of the area have no gravity stations, there is no judgment or control on those interpolation values.

Let u be a continuous function on the real two-dimensionalspace R^2 and let (x_0,y_0) be in R^2. If a set of points

$\{ x_0+a_k , y_0+b_k \}, \qquad k = 1, 2, \ldots, 5$

is also in R^2, then for sufficiently small a_k, b_k and if u has sufficiently many derivatives,

$u_k = u(x_0+a_k , y_0+b_k), \quad k = 1, 2, \ldots, 5$

is approximated by

$$u_0 + a_k\frac{\partial u}{\partial x} + b_k\frac{\partial u}{\partial y} + \frac{1}{2}a_k^2\frac{\partial^2 u}{\partial x^2} + a_k b_k \frac{\partial^2 u}{\partial x \partial y} + \frac{1}{2} b_k^2 \frac{\partial^2 u}{\partial y^2} \qquad [A\text{-}3]$$

To find an expression for $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ at point (x_0,y_0) both sides of equation [A-3] are multiplied by a real number of β_k and a sum is made over k, so that

$$\sum \beta_k u_k = u_0 \sum \beta_k + \frac{\partial u}{\partial x} \sum \beta_k a_k + \frac{\partial u}{\partial y} \sum \beta_k b_k + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} \sum \beta_k a_k^2 + \frac{\partial^2 u}{\partial x\, \partial y} \sum \beta_k a_k b_k + \frac{1}{2}\frac{\partial^2 u}{\partial y^2} \sum \beta_k {b_k}^2 \qquad [A\text{-}4]$$

If β_k are chosen such that

$$\sum \beta_k a_k = 0 \; ; \qquad \sum \beta_k b_k = 0$$
$$\sum \beta_k {a_k}^2 = 0 \; ; \qquad \sum \beta_k a_k b_k = 0$$
$$\sum \beta_k {b_k}^2 = 2 \qquad [A\text{-}5]$$

Then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ at point (x_0,y_0) is approximated by

$$\sum_{k=1}^{5} \beta_k u_k - u_0 \sum_{k=1}^{5} \beta_k \qquad [A\text{-}6]$$

For the present purpose, where one u_k lies off the regular grid, and the remaining four lie on the regular grid, with

a, b=h, 0, - h

a suitable set is

(h,-h), (0,-h), (-h,0), (-h,h), (a_5,b_5),

with $0 \le a_5 < h$ and $0 \le b_5 < h$ (Fig. A.1).

Thus, the expression needed in Eq. [A-1] is :

$$C_{i,j} = \sum_{k=1}^{4} \beta_k u_k - u_{i,j} \sum_{k=1}^{5} \beta_k + \beta_5 u_5 \qquad [A\text{-}7]$$

where $\{u_k\}$ is $u_{i,j-1}$, $u_{i,j+1}$, $u_{i-1,j}$, $u_{i+1,j}$ and u_5 is the *off-grid* point value.

Equation [A-7] is used in [A-2] to give a linear equation between a data point which does not lie on the grid

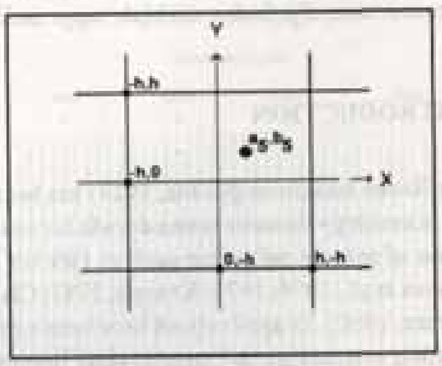

Figure A-1.

point and the other neighbouring grid points, and this equation is called the *off-grid point* equation.