SYNTHETIC SEISMIC RECORDS FROM HETEROGENEOUS MEDIA USING THE RAY TRACING METHOD*

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ABSTRACT

Ray tracing method can be used to generate artificial seismic records from a heterogeneous medium which depend on the structure and stratification of the geologic model. The ray tracing method is a forward modeling process which is able to visualize the seismic wave propagation and its corresponding seismic records from different geologic models, so that the dominant parameters which control the model can be understood.

The ray tracing method divides the heterogeneous medium into cells each having specific velocity. Effectiveness and accuracy using this method need full attention. In general the ray tracing process is slow. This problem was overcome by using eikonal equation which can be derived to yield position, orientation and travel time as a function of the raypath. The accuracy can be overcome by solving the equations for position, orientation and travel time using the high degree polynomial equation.

Experimental results from several geologic models demonstrate the effectiveness of the ray tracing method to visualize the wave propagation in the medium with velocity gradient and complex structures.

I. INTRODUCTION

The forward modeling approach in seismic exploration is a powerful tool for visualizing the seismic wave propagation and its corresponding seismic record from a specific geologic model. With this method the dominant parameters which control a seismic record can be understood. There are three sorts of forward modeling approaches, i.e., the convolutional method, the wave equation method and the ray tracing method (Awali Priyono, 1991).

The convolutional method is the simplest one and is usually used to simulate seismic wave propagations which generate the migrated seismic section. The wave equation method is an accurate method but it consumes too much computer memory and CPU time. The ray tracing method can be considered as the most effective one. Apart from the computational speed which is faster than the wave equation method, this method is accurate in determining the travel time from the source to the receiver (geophone). In addition, it can be expanded to incorporate the seismic wave amplitude in absent of diffraction.

The ray tracing method considers the seismic wave propagation like an optical ray phenomena. This paper stresses its analysis on the kinematical properties of the ray during its propagation along the raypaths in heterogenous media and at the same time computes their travel times. There are two physics laws which were used in the ray tracing method, i.e , the Fermat's principal and the Snell's law. The Fermat's principal deals with the least travel time required by the ray to propagate from the point source to the point detector (Hallyday, 1986). The Snell's law deals with the direction of a ray upon reflection and refraction at an interface.

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To demonstrate the effectiveness of the ray tracing method, several complicated geological models were used as the starting point to generate synthetic seismic records. The geologic models consist of several layers each having velocity gradient either positive or negative. The approach in this paper is not the layer cake model but instead we divided the layers into cells/pixels. The geologic models consist of 100 x 100 cells. Each cell has been assigned to accomodate specific physical quantities (see Figure 1-a)

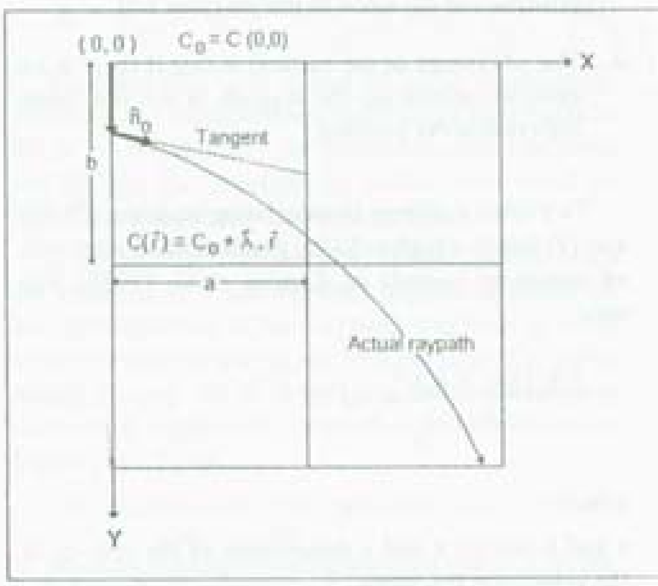


Figure 1-a
The principle of the ray tracing method which is applied in this paper

II. THEORETICAL BACKGROUND

Born and Wolf (1964) introduced a general ray tracing formula which was derived from the eikonal equation. The ray tracing formula is given as :

$$\frac{d}{ds}\left[\frac{1}{c(\vec{r})}\frac{d\vec{r}}{ds}\right] = \nabla_r\left[\frac{1}{c(\vec{r})}\right] \qquad \ldots(1)$$

where :
$c(\vec{r})$ is the ray velocity in the medium at position $\vec{r}$.
s is referred to as affine parameter which is closely related to the length of the raypath (l).

The length of the raypath l is related to the affine parameter by

$$l = \int_0^s |\vec{n}(\vec{r})|\, ds' \qquad \ldots(2)$$

where $\vec{n}(\vec{r}) = \frac{d\vec{r}}{ds}$ is the vector defining the direction of the raypath at position $\vec{r}$.

The travel time of the ray through the raypath is given by

$$t(s) = \int_0^s \frac{|\vec{n}(\vec{r}')|}{c(\vec{r}')} ds' \qquad \ldots(3)$$

If the ray velocity $c(\vec{r})$ is real positive, hence $\vec{n}(\vec{r})$ equal to one and the affine parameter will be identical to the length of the raypath.

Integration of equation (1) twice with respect to s yield

$$\vec{r}(s) = \vec{r_0} + \vec{n_0}\int_0^s \frac{c(\vec{r}')}{c(\vec{r_0})} ds' + \int_0^s c(\vec{r}')\nabla_{r'}\left[\frac{1}{c(\vec{r}'')}\right] ds''ds' \qquad \ldots(4)$$

Equation (4) is an integral equation of a ray at position $\vec{r}$ as a function of the raypath. $\vec{r_0}$ is the position of the source relative to the point of origin (at s = 0) and $\vec{n_0}$ is the unit vector defining the direction of the ray at s = 0.

Equation (4) was solved by considering that the ray travels through a medium with constant velocity gradient,

$$c(\vec{r}) = c_0 + \vec{\lambda}.\vec{r} \qquad \ldots(5)$$

where $\vec{c_0}$ is the velocity of the ray at the origin and $\vec{\lambda}$ is the velocity gradient.

The receprocal velocity given by equation (5) which is referred to as the slowness and can be expanded using the Taylor series of orde 2

$$\frac{1}{c_0+\vec{\lambda}.\vec{r}} = \frac{1}{c_0}\left[1-\left(\frac{\vec{\lambda}.\vec{r}}{c_0}\right)+\left(\frac{\vec{\lambda}.\vec{r}}{c_0}\right)^2\right] \quad \dots (6)$$

in which

$$\left|\frac{\vec{\lambda}.\vec{r}}{c_0}\right| < 1$$

Substitute equation (5) and (6) into equation (4), we obtain

$$\vec{r}(s) = \vec{r_0} + \vec{n_0}s\left[1+\frac{s}{2c_0}(\vec{\lambda}.\vec{n_0})\right] - \frac{\vec{\lambda}s^2}{2c_0} - \frac{\vec{n_0}}{6c_0^2}s^3\,[\vec{\lambda}^2-(\vec{\lambda}.\vec{n_0})^2] \quad \dots (7)$$

$$= -\frac{\vec{n_0}}{6c_0^2}[\vec{\lambda}^2-(\vec{\lambda}.\vec{n_0})^2]s^3 + \frac{1}{2c_0}[\vec{n_0}(\vec{\lambda}.\vec{n_0})-\vec{\lambda}]s^2 + \vec{n_0}s + \vec{r_0}$$

where $\vec{n_0}$ is the unit vector which defines the direction of the ray at s = 0.

The differential of equation (7) with respect to s yield a vector defining the direction of the ray along its path.

$$\vec{n}(s) = \vec{n_0}\left[1+\frac{\vec{\lambda}.\vec{n_0}}{c_0}s\right] - \frac{\vec{\lambda}s}{c_0} - \frac{\vec{n_0}s^2}{2c_0^2}[\vec{\lambda}^2-(\vec{\lambda}.\vec{n_0})^2] \dots$$

(8)

The travel time of the ray along its path can be obtained by substituting equation (6) into equation (3) and by considering that $\vec{n}(\vec{r})$ equal to one. Hence

$$t(s) = \frac{s}{c_0}\left[1+\frac{s^2}{6c_0^2}[\vec{\lambda}^2+(\vec{\lambda}.\vec{n_0})^2] - (\vec{\lambda}.\vec{n_0})\frac{s}{2c_0}\right] \quad \dots (9)$$

It can be concluded from equation (7), (8) and (9) that :

a. The above equations indicate positions, slope and travel time as a function of the raypath.

b. The raypath (s) can be calculated by solving equation (7). To obtain higher accuracy, equation (7) was expanded until degree three and its roots was solved using the Newton-Raphson method. If the priority is in the computational speed, equation (7) can just be expanded till degree two and its roots can be solved by the general quadratic equation.

c. Once the raypath was obtained, its slope and its travel time can be calculated using equation (8) and (9). The position and the slope of the ray which enters the next cell were determined by the position and the slope in the previous cell.

d. The advantage of the method is that if there is an error in calculating the raypath, it did not cause big error in ray position.

To protect accuracy in calculating equation (7), (8) and (9) due to a high velocity gradient at an interface, an empirical formula of Langan et al. (1985) was used.

$$\frac{|\lambda|(a+b)}{c_0} < 0{,}1 \quad \dots (10)$$

where :

a and b are the x and y dimensions of the cell, c_0 is the velocity at the origin.

III. COMPUTATIONAL STRATEGY

Suppose an interface can be defined by a cubic spile interpolation function

$$y = A + Bx + Cx^2 + Dx^3 \quad \dots (11)$$

where A, B, C and D are polynomial constants.

If within the cell in which ray travels there is an interface, the first effort is to calculate the length of the raypath before the ray impinges the interface. The length of the raypath can be obtained by solving the scalar components of equation (7) and substitute them into equation (11). We obtain

$$0=[A+Bx_0+Cx_0^2+Dx_0^3-y_0]+ s[n_{0x} (B+2Cx_0+3D_{0x}^2)-n_{0y}] + s^2 [\alpha (B + 2Cx_0 + 3Dx_0^2) + n_{0x}^2(C + 3Dx_0) - \beta] + s^3 [n_{0x}] [2\alpha(C + 3Dx_0) + Dn_{0x}^2] + s^4 [3D \alpha n_{0x}^2] \quad ..(12)$$

where

$$\alpha = \frac{1}{2c_0}[n_{0x} (\vec{\lambda}.\vec{n_0}) - \lambda_x] \quad ... (13)$$

and

$$\beta = \frac{1}{2c_0} [n_{0y} (\vec{\lambda} . \vec{n_o}) - \lambda_y] \quad ... (14)$$

Equation (12) was solved by using the Newton-Raphson and the result was substituted into equation (7) to get the absica of the intersection point between the ray and the interface. Its ordinat was found by substituting the absica into equation (11).

If ray impinges an interface, the intersection of the ray with the interface becomes a point of origin for the new equation of the raypath. Suppose ψ is the angle between incidence ray with the normal of an interface having dip θ, while γ is the angle between transmitted ray with the normal of this interface (see Figure 1-b). Then :

The x-component of the incidence ray is

$$N_{rx}= \frac{1}{1+tan^2\theta}[-\sin(\psi-\theta) \tan^2\theta+ 2\cos(\psi-\theta)\tan\theta+ \sin(\psi-\theta)]$$

The y-component of the incidence ray is

$$N_{ry}= \frac{1}{1+\tan^2\theta}[\cos (\psi -\theta)\tan^2\theta+2\sin(\psi-\theta)\tan\theta\cos(\psi-\theta)]$$

The x-component of the transmitted ray is

$$N_{tx} = \frac{1}{\sqrt{1+\tan^2\theta}} [\sin \gamma - \tan\theta \sqrt{1- \sin^2\gamma}]$$

The y-component of the transmitted ray is

$$N_{ty} = \frac{1}{\sqrt{1 + \tan^2\theta}} [\sqrt{1 \ \sin^2\gamma} + \sin \gamma \tan \theta]$$

The ray tracing method applied for seismic forward modeling of the heterogeneous geological structure can be carried out using the following steps :

1. Construct the geologic model and assume that the physical properties of the medium can be divided into cells/pixels, either square or triangle.
2. Determine the origin of the ray (source coordinate), the departure angle of the ray (from which can be determined the unit vector defining the direction of the ray) and the coordinates of the receivers.
3. Check whether within the cell there is an interface.
4. If there is no interface, determine the shortest length of the raypath to the cell's edge, the coordinate of the intersection point with the cell's edge, the travel time of the ray to the cell's edge and vector which defines the direction of the new ray. The coordinate of the intersection point between the ray and the cell's edge becomes the position of the new ray for the next cell.

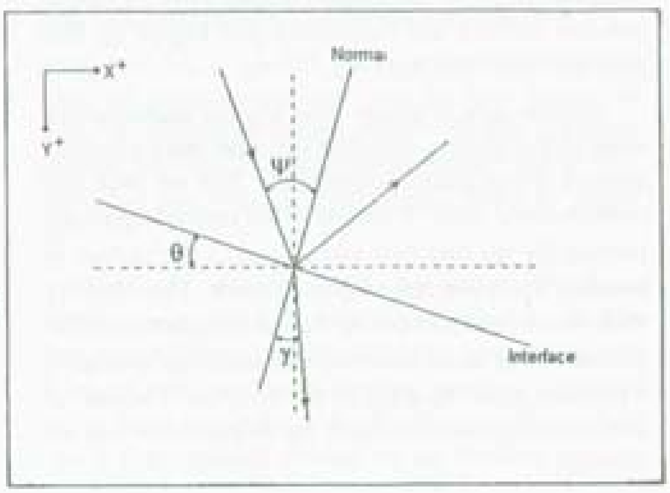


Figure 1-b
Ray having angle of incidence ψ will be reflected and transmitted by an interface whose dip angle is θ with respect to x axis

5. If there is an interface, calculate the length of the raypath to the cell's edge and compute the length of the raypath to this interface. Compare both lengths. If the length of the raypath to the cell's edge is shorter than the length of the raypath to the interface, do step 4. If contrary, the ray will be reflected and transmitted by the interface. Calculate the coordinate of the intersection point between ray and the interface, the travel time of the ray to the interface and vector which defines the direction of the new ray. The coordinate of the intersection point between the ray and the cell's edge becomes the position of the new ray for the next cell. Back to step 3.

6. Determine the coordinate of the ray at the earth's surface (zero depth). Calculate the distance between that point to the nearest receiver. If the distance is equal to the defined distance between source and receiver plus its tolerance, then the ray tracing process is completed. Otherwise, return to step 1 with a new shooting angle.

In step 6, the smaller the increment of the shooting angle, the better the result, but it consumes too much computer time. To reduce the computer time, an interpolation method was introduced (see Figure 2). The principal is as follows :

Assume rays originate from a point source which were shot regularly one after the other with a regular interval $\Delta\Phi$ degrees. Suppose the first ray was shot with shooting angle $\Phi_1$degree. This ray undergoes reflection by the interface and arrives at the surface at position x_1 to the left of the receiver. The next ray with the shooting angle of $\Phi_1 + \Delta\Phi$ degrees was reflected by the same interface and reach the surface at a position x_2 to the right of the receiver. The correct shooting angle which causes ray to arrive just at a receiver is :

$$\Phi_b = \Phi_1 + (x_p - x_1)\,\frac{\Delta\Phi}{\Delta x} \qquad \ldots (15)$$

where :

x_p is the distance from the source to the receiver, Δx is the distance between x_1 and x_2.

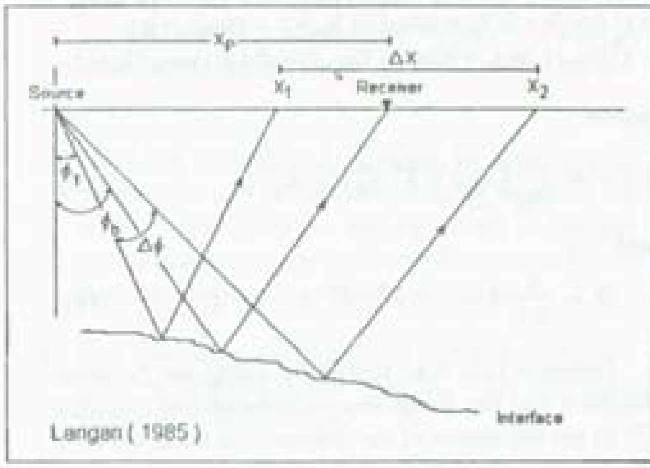


Figure 2
Interpolation method for determining the correct shooting angle so that ray originates from the source will reach geophone on the surface

IV. RESULTS AND DISCUSSION

Figure 3-a is a geologic model which consists of two layers. In the first layer there is a velocity gradient as can be seen in Figure 3-b. In the second layer the velocity is constant. The source was located at the surface whose position is (8000,0). The unit length is feet. We want to know the pattern of the seismic record which may come out from this specific geologic model if the geophone was planted on the surface from position (0,0) to position (25000,0) with geophone interval of 120 feet.

Figure 3-c is the result of the ray tracing using the concept described in this paper. The corresponding synthetic seismic record is illustrated in Figure 3-d. Events A and B are the direct waves. The curvature of the rays are due to the velocity gradient. Events C are the reflections from the dome of the structure. The depths of the reflecting points vary between 3000 feet

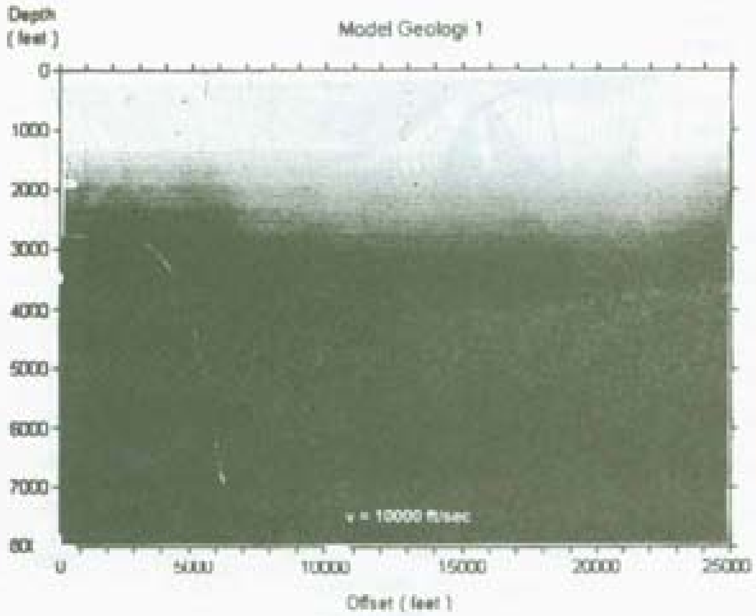


Figure 3-a
A geologic model with consists of two layers. A positive velocity gradient was assigned in the first layer (see Figure 3-b)

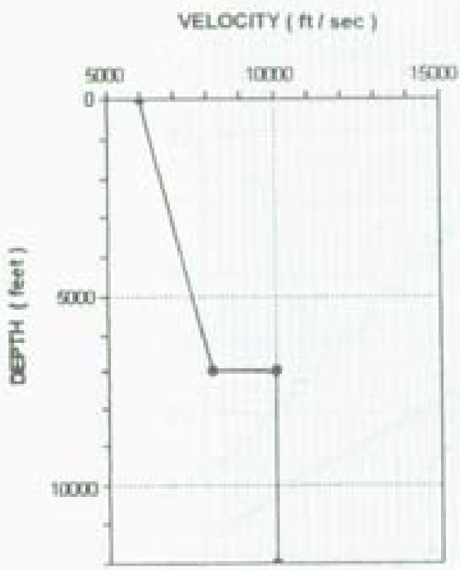


Figure 3-b
The velocity profile of the geologic model shown in Figure 3-a

to 5000 feet. Events D signify reflections from the flat part of the structure whose depths are 7000 feet.

Figure 4-a is another geologic model which is a little bit more complicated than the first model. Its velocity log can be seen in Figure 4-b. There are two positive velocity gradients, one negative velocity gradient and one velocity contrast at a depth of 90000 feet.

The result of ray tracing is illustrated in Figure 4-c. The corresponding synthetic record can be seen in Figure 4-d. Events A are direct wave, events B are reflections from the down dipping interface, events C are reflections from the up dipping interface, while events D are reflections from the flat interface.

Figure 5-a is a geologic model which consists of an undulating interface with positive velocity gradient in the layers above and below it. The source was located at (12000,0). The receiver was arranged to

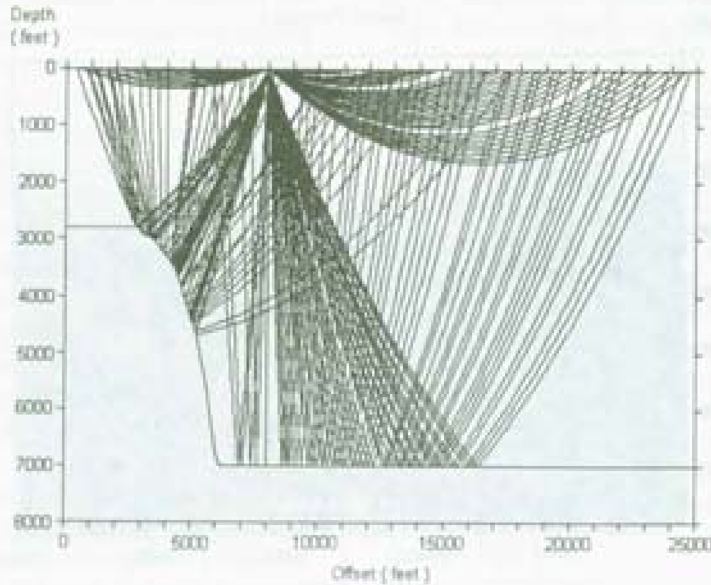


Figure 3-c

The tracing of the rays in the medium described by the geologic model Figure 3-a

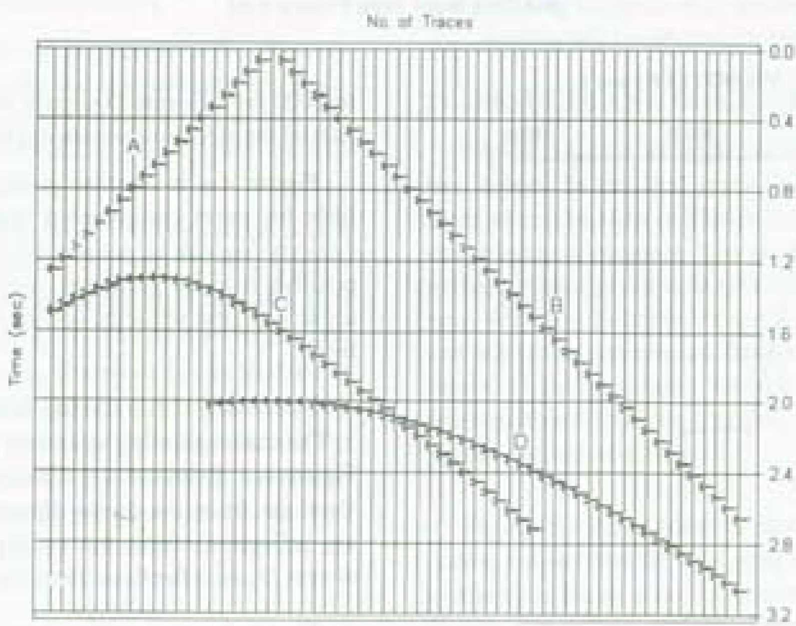
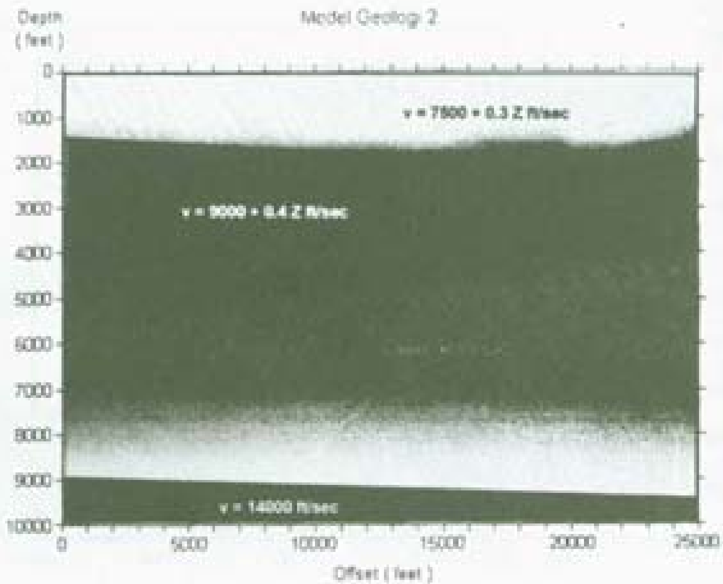


Figure 3-d

The resulting synthetic seismic record associated with the geologic model Figure 3-a and the raypaths in Figure 3-c



Fugure 4-a
A structurally more complicated geologic model than Figure 3-a. The velocity gradients were assigned in layer 1, 2 and 3

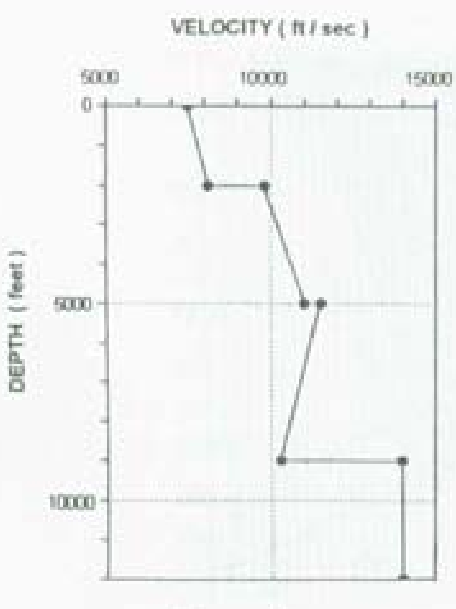


Figure 4-b
The velocity profile of the geologic model shown in Figure 4-a

spread between (0,0) up to (25000,0) with an interval of 120 feet. The result of ray tracing can be seen in Figure 5-c and the corresponding synthetic record is illustrated in Figure 5-d. Events A dan B are the direct waves. Events C are reflections from the undulating interface. Only a small part of this interface can be recorded using the given receiver spread. Events D are reflections from the flat interface which is located below the undulating interface.

Figure 6-a looks like a simple geologic model, but it consists of two layers with positive velocity gradients and a layer with negative velocity gradient as can be seen in Figure 6-b. It was intended to demonstrate the bending of the rays (see Figure 6-c). Compare the curvature of the rays in the layer with negative velocity gradient and layer with positive velocity gradient. Figure 6-d is the synthetic record corresponding to the ray tracing process illustrated in Figure 6-c. Events A are the direct waves, events B

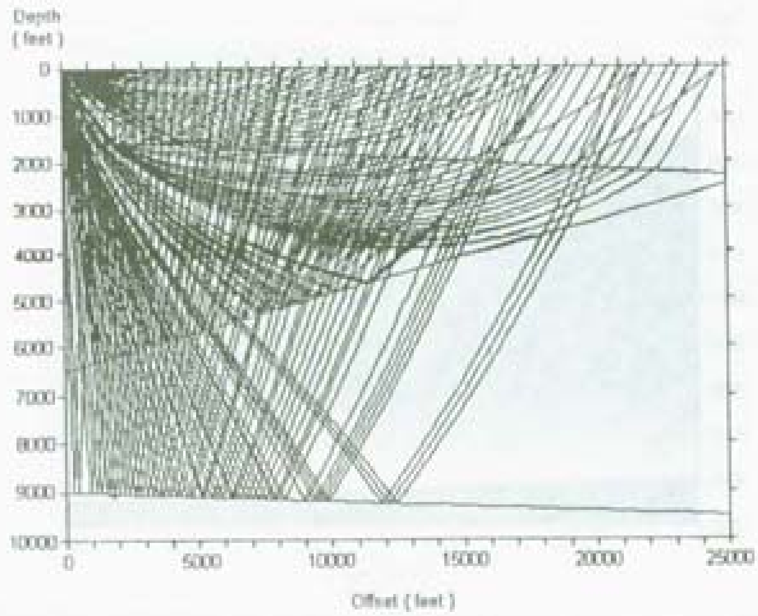


Figure 4-c
The tracing of rays in the medium constrained by the structure as shown in Figure 4-c

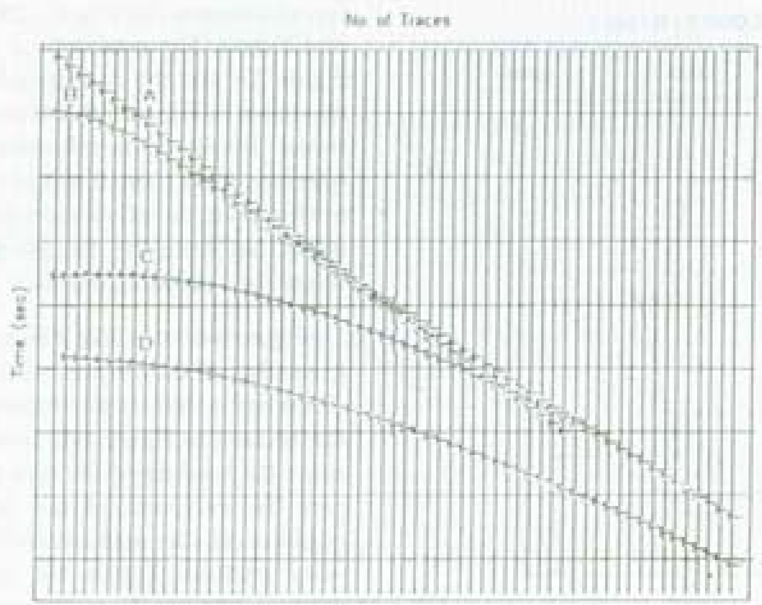


Figure 4-d
The synthetic record corresponding to geologic model of Figure 4-a and raypaths of Figure 4-c

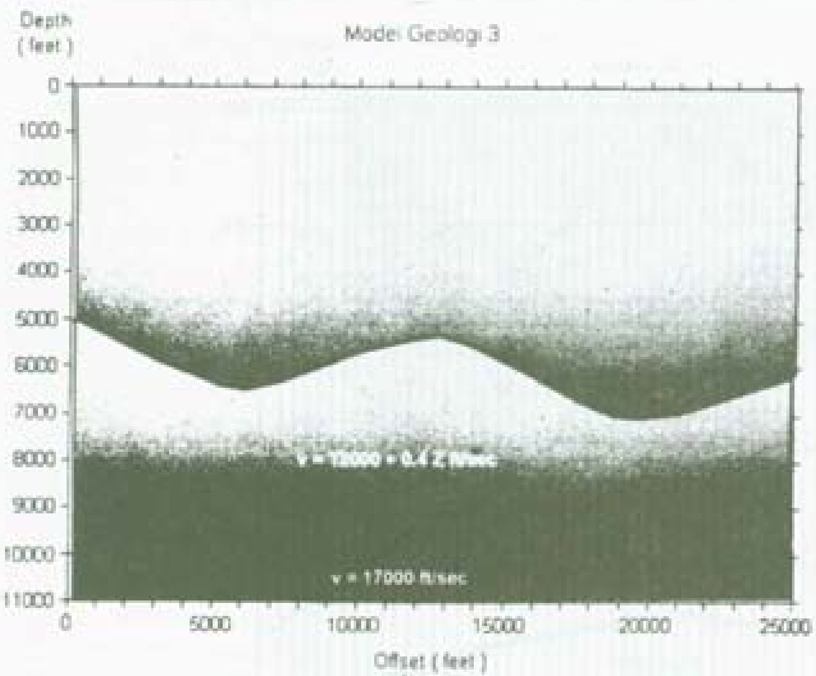


Figure 5-a
A geologic model which includes an undulating interface and velocity gradient above and below it

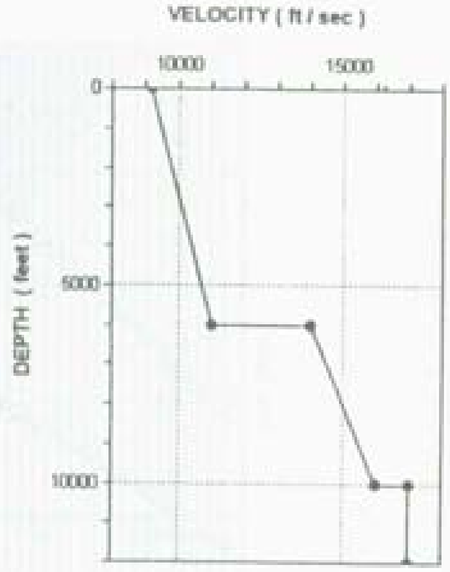


Figure 5-b
The velocity profile corresponding to the geologic model of Figure 5-a

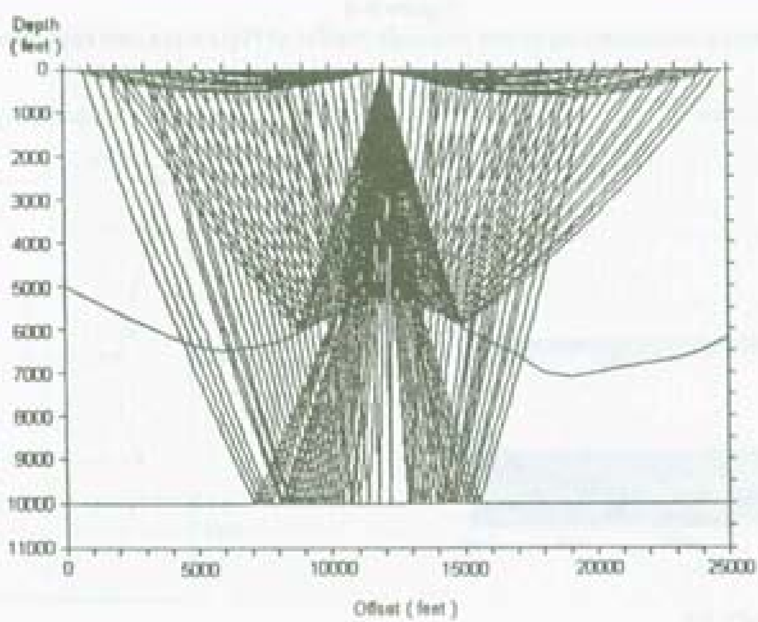


Figure 5-c
The result of ray tracing in the medium described by the geologic model in Figure 5-a

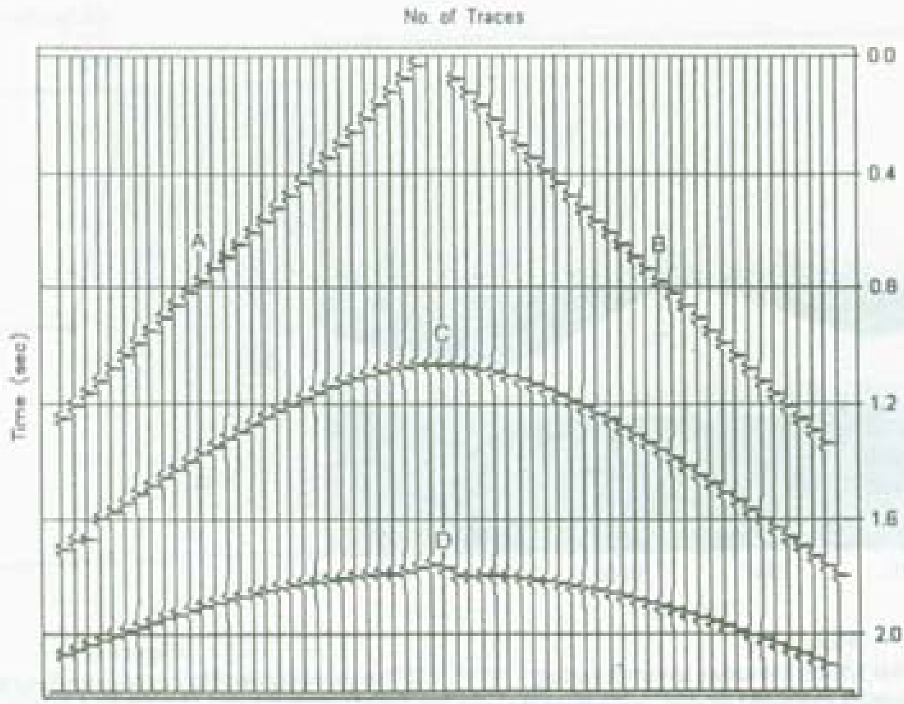


Figure 5-d
The synthetic record corresponding to the geologic model of Figure 5-a and raypaths of Figure 5-c

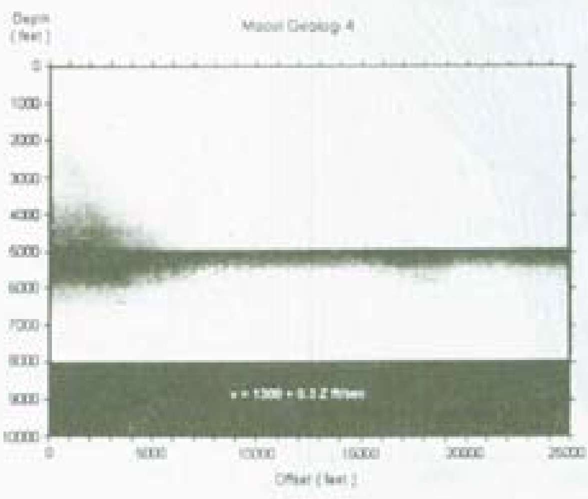


Figure 6-a
A geologic model which consists of three layers having velocity gradient. The negative velocity gradient was assigned for the middle layer

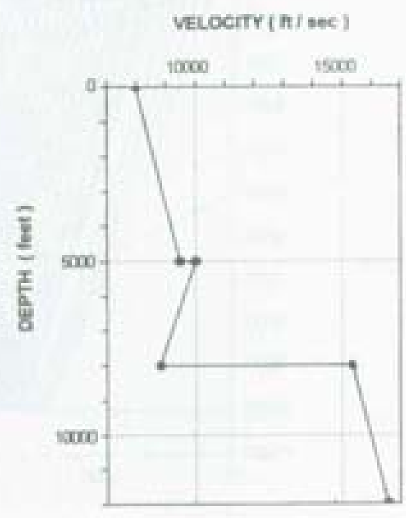


Figure 6-b
The velocity frofile corresponding to the geologic model shown in Figure 6-a

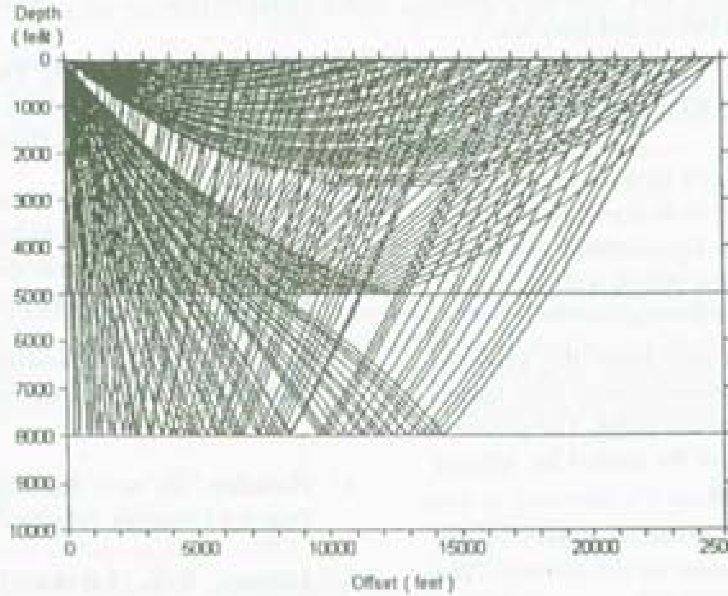


Figure 6-c
The tracing of the ray in the medium described by the geologic model given in Figure 6-a. See the bending of the rays in the medium having positive velocity gradient and negative velocity gradient

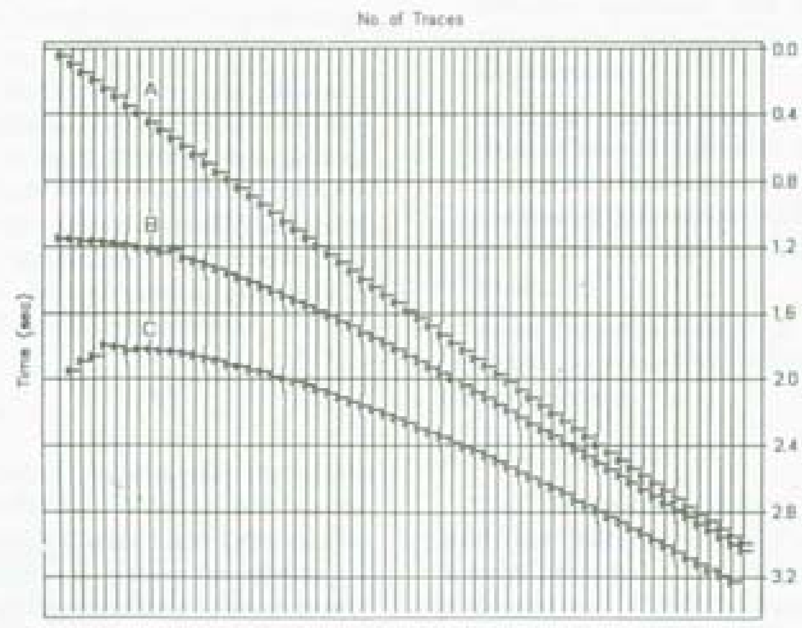


Figure 6-d
The synthetic record corresponding to the geologic model shown in Figure 6-a and the raypaths illustrated in Figure 6-c

are the reflections from the first interface, while events C are reflection from the second interface.

V. CONCLUSIONS AND RECOMMENDATION

1. The ray tracing method as a forward modeling approach is very useful for understanding and interpreting seismic record. This method can easily generate synthetic seismic record with higher accuracy from complicated geologic model.
2. The computational process from the geologic model to the synthetic seismic record does not require specific high speed computer. The common PC series 486 available in the market has already sufficient for this purpose.
3. This paper stresses its modeling aspect on the travel time from the source to the receiver. The variation of amplitude and reflection coefficient as a function of offset distance is recommended for the next research work.

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