A SIMPLE APPROACH FOR UNDERSTANDING SEISMIC WAVE PROPAGATION IN POROUS MEDIA

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ABSTRACT

A simple approach for understanding seismic wave propagation in porous media has been developed based on the effect of stress gradient on compressibility of the matrix and the fluid. The fluid saturation is accommodated in the bulk density formulation. The approach started from Gassmann theoretical formulation followed by simplifying the mathematical detail by substituting their physical aspects. Finally a practical formula for core analysis purposes is introduced. The theoretical concepts and experimental results appear to be in a good agreement.

I. INTRODUCTION

The propagation of seismic waves in subsurface rocks appears to play a very important role in collating information from subsurface media. A thorough understanding of its mechanism will be very helpful in revealing the predominant petrophysical factors which control the wave propagation. The efforts to search theoretical backgrounds on this subject have been continuously carried out along with various experimental works.

The first theoretical approach which considers subsurface rock as a solid material, was proposed at the beginning of the twentieth century, had been proven not sufficient to resolve the problems occurred in porous media such as reservoir rock. Wyllie et al (1956) have proposed a practical approach, i.e. the so-called time-average relation for fluid-saturated porous material. In this approach the porous rock was replaced by a series of alternating solid and liquid layers with the wavefront passes perpendicular to the interfaces between both phases. Although Wyllie's approach is better than the first one, however, it is still over-simplifying the reality. The time-average relation entirely neglects the important role of the bulk deformation properties of the rock material, which are pressure dependent (Geerstma and Smit, 1961).

The approach presented in this paper assumes that the reservoir rock consists of matrix which form skeleton, pore spaces and fluid, each of them has compressibility and density. The propagation of seismic wave through the porous media will change its petrophysical properties instantaneously. Initially our approach was in accordance with Gassmann (1951), Biot (1956) and Geerstma and Smit (1961), however due to the complicated mathematical formulations found in their papers, we have decided to simplify its explanation. A similar method has also been used by White and Sengbush (1987) to deal with this problem. A series of experimental works with cores have been incorporated to our approach in order to prove its validity. Nefrizal and Munadi (1997) have also used similar approach with an emphasis on numerical modeling.

II. THEORETICAL APPROACH

A. *The Classical Concept*

The classical approach for understanding the seismic wave propagation in subsurface rock is based upon the assumption that subsurface rock is homogeneous, isotropic and perfectly elastic. The elasticity theorem and the Newton's second law of motion were used to derive equation of motion of the particle medium under the influence of an applied force (mechanical disturbance). This equation of motion led to the well known wave equation which is able to predict the existence of two body waves propagating in the medium i.e. the longitudinal or P-wave and the transverse or S-wave. A more detailed explanation of this approach can be found in Morgan (1950), Ewing et al (1957), Howell (1959), Kolsky (1963), Sheriff and Geldart (1982).

The P-wave velocity derived from the above mentioned approach, is given by :

$$V_p = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho_b}} \qquad \text{(1a)}$$

and the S-wave velocity is calculated using :

$$V_s = \sqrt{\frac{\mu}{\rho_b}} \qquad \text{(1b)}$$

where:

k is the bulk modulus of the rock

μ is the rigidity

ρ_b is the bulk density.

Our approach is also based on equation (1), however, we have modified the factors which are affected by the existence of pore space and fluid, i.e. k and ρ_b.

B. *The Porous Media Concept*

1. Bulk density as a function of porosity, fluid density and matrix density

For porous media containing fluid, the bulk density will be of the form:

$$\rho_b = \phi\, \rho_f + (1-\phi)\, \rho_m \qquad \text{(2a)}$$

where

ρ_b is the bulk density of the rock

ϕ is the porosity

ρ_f is the density of the fluid

ρ_m is the density of the matrix.

If the fluid consists of water and hydrocarbon, equation (2a) can be expanded to become:

$$\rho_b = \phi \left\{ S_w \rho_w + (1-S_w)\, \rho_h \right\} + (1-\phi)\, \rho_m \qquad \text{(2b)}$$

where :

ρ_w is the density of water

ρ_h is density of hydrocarbon

S_w is the water saturation.

2. Bulk modulus of the fluid

The bulk modulus of porous rock which contains fluid should be split into two parts, i.e. the bulk modulus of the matrix k_m and the bulk modulus of the fluid k_f.

The bulk modulus of the fluid k_f can be defined as the reciprocal of the fractional change in fluid volume ($\Delta V_f/V_f$) due to a small change of the pore pressure (ΔP_f),

$$k_f = \frac{\Delta P_f}{\Delta V_f / V_f} \qquad \text{(3)}$$

since $V_f = \phi V$ (V is the bulk volume or the total volume)

$$k_f = \frac{\Delta P_f . \phi . V}{\Delta V_f}$$

or

$$\Delta V_f = \frac{\Delta P_f . \phi . V}{k_f} \qquad \text{(4)}$$

Equation (4) expresses the small change of the fluid volume due to a small change of the fluid pressure.

3. Bulk modulus of the matrix

The bulk modulus of the matrix by analogy to equation (3) can be formulated as,

$$k_m = -\frac{\Delta \bar{P}}{\Delta V_{m1} / V} \qquad \text{(5)}$$

where :

$\Delta \bar{P}$ is the pressure applied to reservoir rock under investigation when the rock is dry

$\Delta V_{m1}/V$ is the fractional change of the matrix volume due to the pressure applied on it

V is the bulk volume.

Due to reaction force of the fluid (ΔP_f) against an external pressure $\Delta \bar{P}$, the matrix also undergoes a small change in volume (ΔV_{m2}), so that the bulk modulus can also be written as

$$k_m = -\frac{\Delta P_f}{\Delta V_{m2} / V_m} \qquad \text{(6)}$$

Since $V_m = (1 - \phi)\, V$ equation (6) can be rewritten as

$$\Delta V_{m2} = -\frac{\Delta P_f (1-\phi) V}{k_m} \qquad \text{(7)}$$

The total volume change due to the stress acting on a porous rock is the sum of three factors given in equation (5), equation (7) and equation (4)

$$\Delta V = \Delta V_{m1} + \Delta V_{m2} + \Delta V_f \qquad \text{(8)}$$

or

$$\frac{\Delta V}{V} = -\frac{1}{k_m} \Delta \bar{P} - \left(\frac{(1-\phi)}{k_m} + \frac{\phi}{k_f} \right) \Delta P_f \qquad \text{(9)}$$

4. Bulk modulus of the dry rock

When the rock is dry, it means that the pores are empty, the stress $\Delta\bar{P}$, acting on it will cause a fractional change in volume with respect to the bulk volume ($\Delta V_1/V$). In this condition we use k_d to express the bulk modulus of the dry rock,

$$k_d = -\frac{\Delta\bar{P}}{\Delta\bar{V}_1/V} \quad (10)$$

or

$$\frac{\Delta\bar{V}_1}{V} = -\frac{\Delta\bar{P}}{k_d} \quad (11)$$

When the porous rock is saturated by water or hydrocarbon the reaction force from the fluid (ΔP_f) will also cause a fractional change in volume with respect to the total volume $\left(\Delta\bar{V}_2/V\right)$. The bulk modulus of the rock (matrix) can be written as

$$k_m = -\frac{\Delta P_f}{\Delta\bar{V}_2/V} \quad (12)$$

or

$$\frac{\Delta\bar{V}_2}{V} = -\frac{\Delta P_f}{k_m} \quad (13)$$

The total volume change in this situation can be formulated as

$$\frac{\Delta V}{V} = \frac{\Delta\bar{V}_1}{V} + \frac{\Delta\bar{V}_2}{V} \quad (14)$$

Substituting equation (11) and equation (13) into equation (14) yields,

$$\frac{\Delta V}{V} = -\frac{\Delta\bar{P}}{k_d} - \frac{\Delta P_f}{k_m} \quad (15)$$

5. Bulk modulus of the porous saturated rock

The bulk modulus of the porous rock as a system which can be as a dry state or as a saturated state is usually written as

$$k = -\frac{\Delta P}{\Delta V/V} = -\frac{\Delta\bar{P} + \Delta P_f}{\Delta V/V} \quad (16)$$

Equation (16) will be best if it can be formulated by using the equations derived in the previous sub paragraph.

We recall that $\Delta V/V$ from equation (9) and equation (15) should be the same, while the relationship between $\Delta\bar{P}$ and ΔP_f and k_d, k_f, k_m is not clear.

To overcome the problem, let us multiply equation (9) by $1/k_d$ and equation (15) by $1/k_m$ and rewrite and substract as follows

$$\frac{\Delta V}{V}\cdot\frac{1}{k_d} = -\frac{1}{K_d K_m}\Delta\bar{P} - \left(\frac{1-\phi}{K_d K_m} - \frac{\phi}{K_d K_f}\right)\Delta P_f \quad (17)$$

$$\frac{\Delta V}{V}\cdot\frac{1}{k_m} = -\frac{1}{k_d k_m}\Delta\bar{P} - \frac{\Delta P_f}{k_m^2} \quad (18)$$

$$\frac{\Delta V}{V}\left(\frac{1}{k_d} - \frac{1}{k_m}\right) = \left(\frac{1}{k_m^2} - \frac{\phi}{k_f k_d} - \frac{1-\phi}{k_m k_d}\right)\Delta P_f \quad (19)$$

it can be followed that

$$\Delta P_f = \frac{\dfrac{\Delta V}{V}\left(\dfrac{1}{k_d} - \dfrac{1}{k_m}\right)}{\left(\dfrac{1}{k_m^2} - \dfrac{\phi}{k_f k_d} - \dfrac{1-\varphi}{k_m k_d}\right)} \quad (20)$$

$$\Delta\bar{P} = -\left(k_d\frac{\Delta V}{V} + k_d\frac{\Delta P_f}{k_m}\right)$$

$$= \frac{\Delta V}{V}\left[-k_d - \frac{\dfrac{k_d}{k_m}\left(\dfrac{1}{k_d} - \dfrac{1}{k_m}\right)}{\left(\dfrac{1}{k_m^2} - \dfrac{1}{k_f k_d} - \dfrac{1-\phi}{k_m k_d}\right)}\right] \quad (21)$$

Substitute equation (20) and equation (21) into equation (16), leads to

$$k = k_d + \frac{k_d\left(1 - \frac{k_d}{k_m}\right)\left(\frac{1}{k_d} - \frac{1}{k_m}\right)}{\left(\frac{1-\phi}{k_m} + \frac{\phi}{k_f} - \frac{k_d}{k_m^2}\right)} \quad ..(22)$$

or

$$k = k_d + \frac{\left(1 - \frac{k_d}{k_m}\right)^2}{\frac{1-\phi}{k_m} + \frac{\phi}{k_f} - \frac{k_d}{k_m^2}} \quad ......(23)$$

or

$$k = k_d + k_f \frac{\left(1 - \frac{k_d}{k_m}\right)^2}{\left(1 - \frac{k_f}{k_m}\right)\phi + \left(k_m - k_d\right)\frac{k_f}{k_m^2}} \quad .(24)$$

C. *The Elastic Wave Velocities in Porous Media*

By substituting equation (24) into equation (1.a) we obtain the P-wave velocity in porous media:

$$V_p = \left[\left\{\left(k_d + \tfrac{4}{3}\mu\right) + \frac{k_f\left(1 - \frac{k_d}{k_m}\right)^2}{\left(1 - \frac{k_f}{k_m}\right)\phi + \left(k_m - k_d\right)\frac{k_f}{k_m^2}}\right\}\frac{1}{\rho_b}\right]^{1/2} \quad (25)$$

$$kf = \frac{1}{S_w C_w + (1 - S_w) C_h}$$

where :

S_w is the saturation of water/brine

C_w is the compressibility of water

C_h is the compressibility of hydrocarbon.

This equation tells us that the P-wave velocity in porous media containing fluid are the functions of compressibility of the dry rock, porosity, rigidity, compressibility of the fluid, compressibility of the matrix and bulk density.

The S-wave velocity in porous media can also be formulated by substituting equation (2a) into equation (1b) as

$$V_s = \left[\frac{\mu}{\phi\rho_f + (1-\phi)\rho_m}\right]^{1/2} \quad ......(26)$$

It can be easily understood that the S-wave velocity in porous media depends on the porosity, rigidity, density of the fluid and density of the matrix.

If we substitute $k_d = \beta/C_m$, $Cm = 1/km$ and $C_f = 1/k_f$ into equation (25) we will obtain equation similar to Gassmann's (1951) and Geerstma's (1961) formula as formulated later on by Domenico (1977),

$$V_p = \left[\left\{\left(\frac{\beta}{C_m} + \frac{4}{3}\mu\right) + \frac{(1-\beta)^2}{(1-\beta-\phi)C_m + \phi C_f}\right\}\frac{1}{\rho_b}\right]^{1/2} \quad (27)$$

where

C_m is the compressibility of the matrix

C_f is the compressibility of the fluid.

Let C_b be the compressibility of reservoir rock, which Geerstma and Smit assumed that $C_b = 1/k_d$. This is correct if the rock is dry or contains gas. If the rock contains oil then, $k_d^{\ 1} = 1/C_b$. In this case Gassmann's (1951) formula is more appropriate.

III. CHECKING METHODOLOGY

To test the effectiveness of our theoretical approach, a series of experimental works have been done, and the results are plotted on the same graph with those of theoretical works.

The experimental works consist of placing a core of reservoir rock in a sample holder in which pressure can be adjusted. An electronic device for measuring the travel time of the P-wave and S-wave was also incorporated in this system. More detailed explanation of the experimental set up as well as calculation methods used to provide theoretical values, can be seen in Widarsono and Saptono (1997). For measuring bulk density, porosity and matrix and fluid compressibilities the standard measurement tools were used.

It becomes evident that equation (25) is easier to be implemented than equation (27), because k_d can be obtained by measuring V_p and V_s whilst the remaining factors can be measured directly.

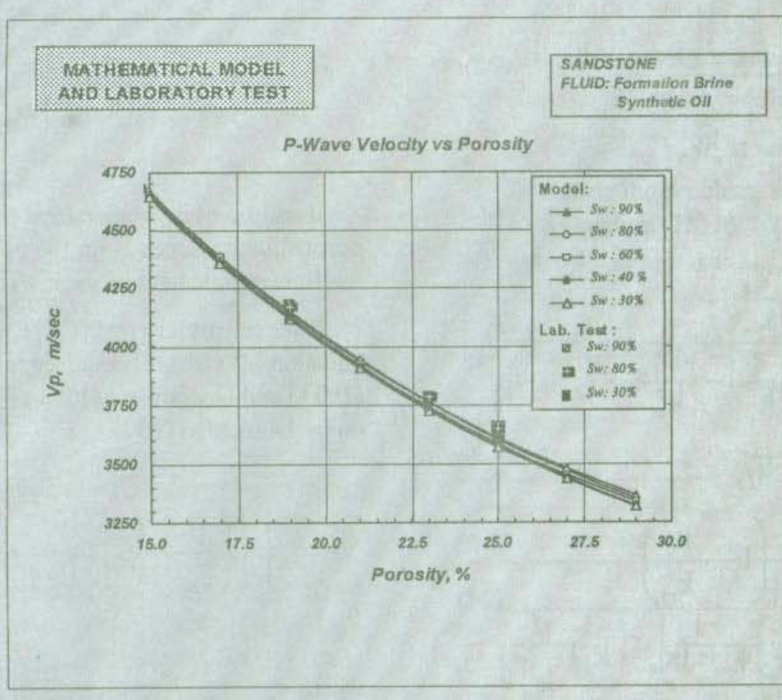

Figure 1

P-wave velocity as a function of porosity with different values of water saturation It is obvious that the increase of porosity causes the decrease of P-wave velocity

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. *P-wave Velocity as a Function of Porosity*

Three samples of clean sandstone saturated by brine and synthetic oil were examined. The water saturation in the samples were 90%, 80%, and 30% respectively, and their porosities were measured along with their compressibilities, rigidities and bulk modulus consistent with equation (25). The results of those experiments are illustrated in Figure-1. As expected, the P-wave velocity decreases as the porosity increases. The relationship is not linear. Computed model based on the theoretical formula and the results obtained from experiments exhibit a good agreement.

B. *P-wave Velocity as a Function of Water Saturation*

Equation (25) can be used as a reference for an experimental work which relates P-wave velocity and water saturation by replacing ρ_b with factors affecting bulk density in saturated porous rock as given in equation (2b). Using the same samples which were used in the previous experiment we obtain P-wave velocity as a function of water saturation for different values of porosity, i,e., 25%, 23% and 19% respectively (Figure-2). It can be seen that theoretical results (model) and experimental results (lab test) are in a good agreement. The P-wave velocity increases slightly as the water saturation increases.

C. *P-wave Velocity as a Function of Fluid Modulus*

Equation (25) can be easily used to guide an experiment which explains the effect of fluid compressibility on P-wave velocity. Using the same samples which were used in the first experiment but changing the fluid content, we obtained the results as presented in Figure-3. The P-wave velocity increases slightly as the bulk modulus of the fluid increases. Again, experiment and theorem display similar results.

D. *P-wave Velocity as a Function of Bulk Modulus of The Saturated Rock*

The bulk modulus of saturated rock (k) consists of the bulk modulus of the dry rock (k_d), the bulk modulus of the matrix (k_m) and the bulk modulus of the fluid (k_f).

Equation (25) enables us to reveal the relationship between P-wave velocity and the bulk modulus of saturated rock for different porosity and water saturation. Results of

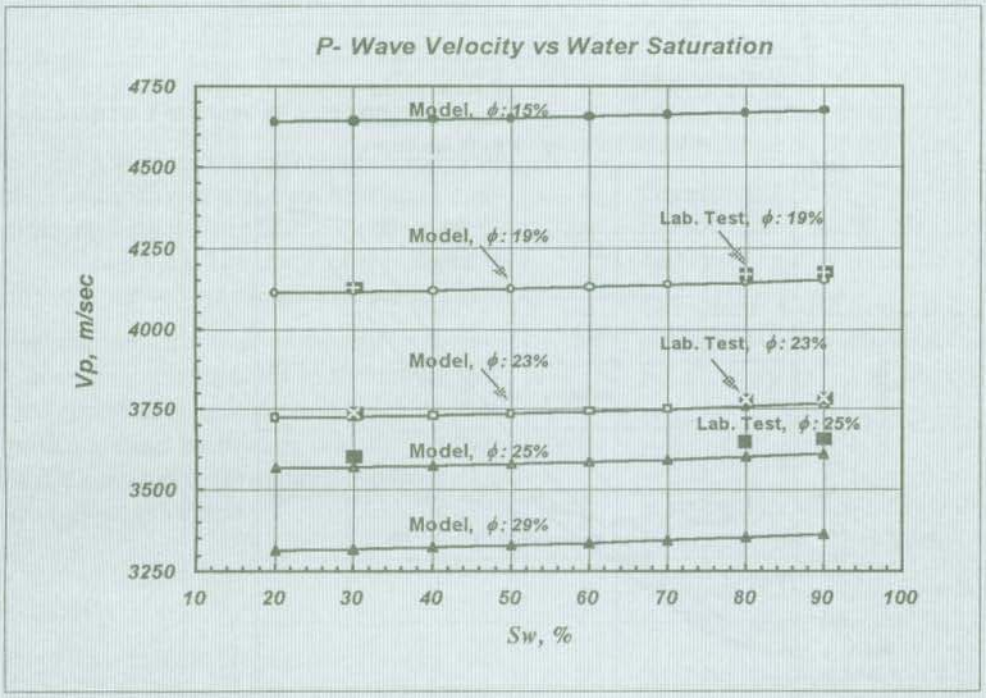

Figure 2
P-wave velocity as a function of water saturation with different values of porosity
It seems that the effect of water saturation on P-wave velocity is not significant

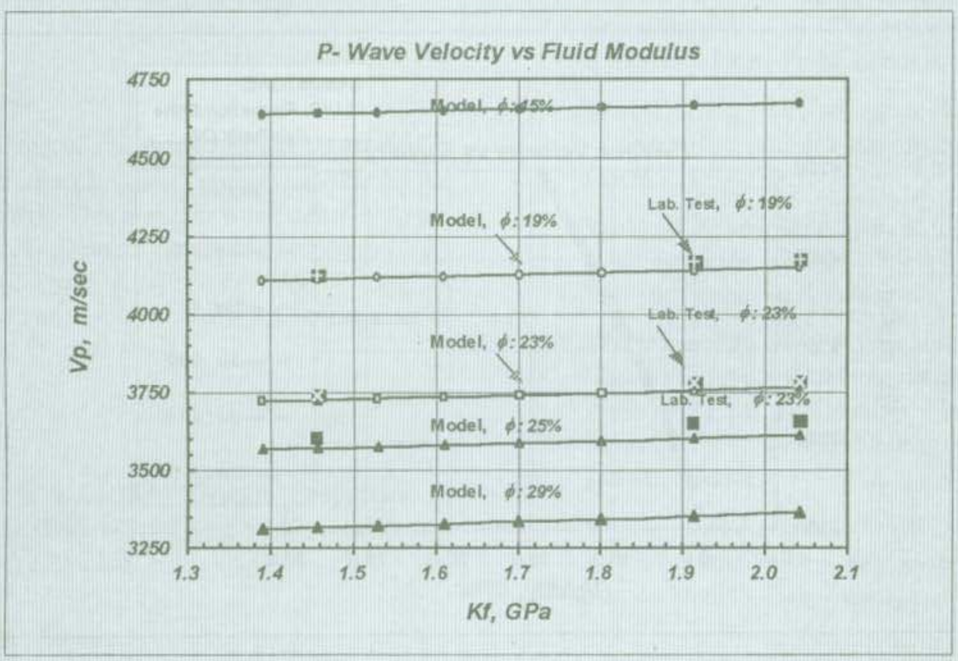

Figure 3
P-wave velocity as a function of bulk modulus of the fluid with different values of porosity
It seems that the fluid modulus has a very little effect on P-wave velocity

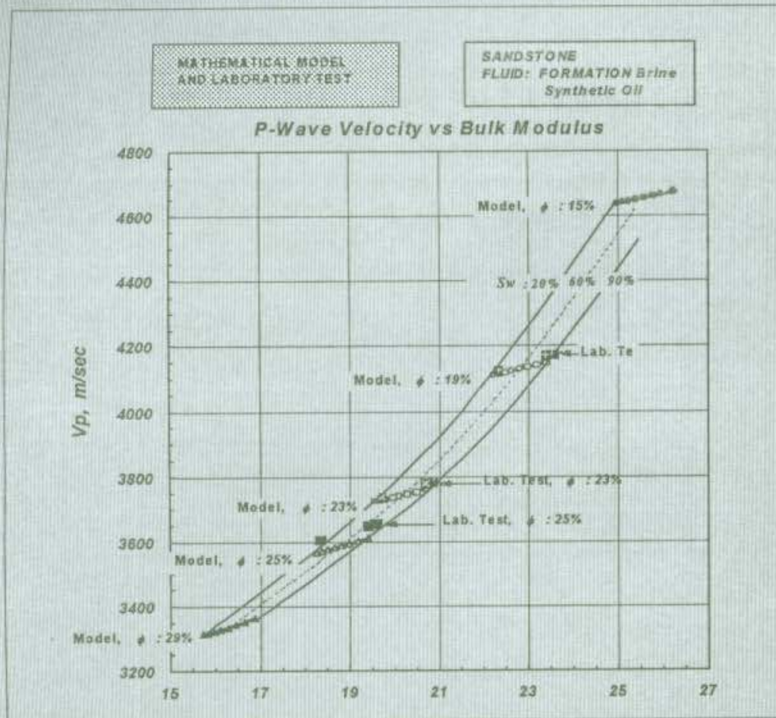

Figure 4

P-wave velocity as a function of bulk modulus of the porous rock with different values of porosity.The effect of the bulk modulus of the rock is significant

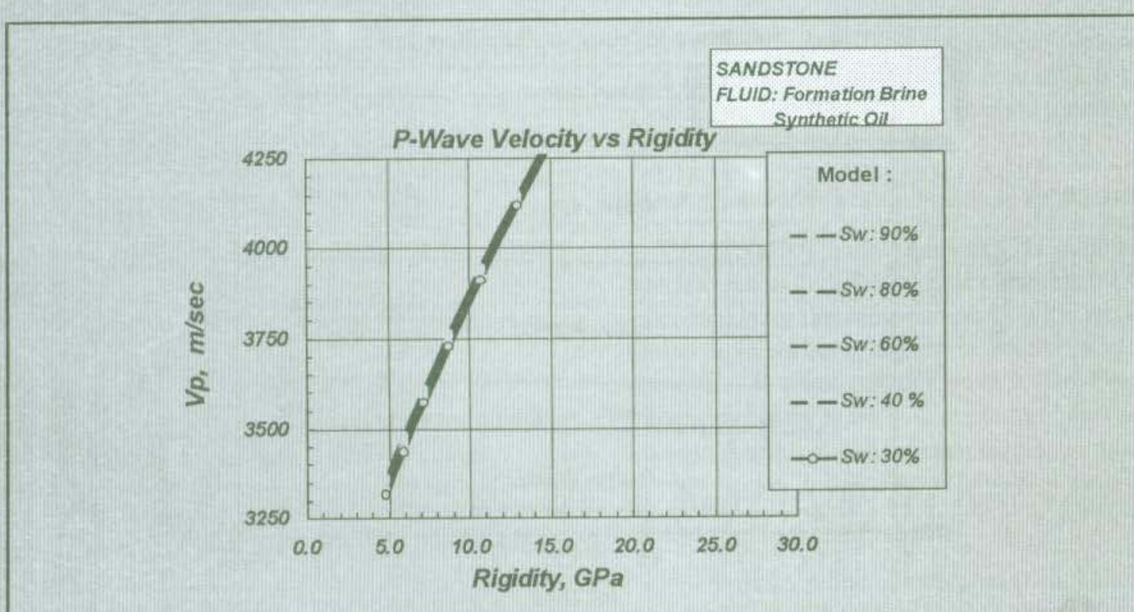

Figure 5

P-wave velocity as a function of rigidity with different values of water saturation

The effect of rigidity on P-wave velocity is very significant the experiments are given in Figure-4. It can be seen again that experiments and theorem demonstrate a good agreement.

E. *P-wave Velocity as a Function of Rigidity*

The rigidity of a porous rock is different to the rigidity of a solid rock. The rigidity of a porous rock can be obtained by multiplying bulk density with the square of S-wave velocity (Equation 1.b). The porosity is included in the bulk density, so is the water saturation (Equation 2).

Figure-5 demonstrates the effect of rigidity on P-wave velocity of a porous saturated rock. It can be seen that water saturation has no effect on that curve but rigidity itself has a very significant effect on P-wave velocity. Contrary to porosity and bulk modulus, the rigidity apparently has strongest effect on P-wave velocity.

V. CONCLUSION

The velocity of seismic wave propagation in porous rock which contains fluid depends on the porosity, bulk density, compressibility of matrix, compressibility of fluid, compressibility of dry rock and the rigidity.

The effect of porosity and bulk modulus on P-wave velocity is much more significant compared to the effect of water saturation or fluid compressibility, where as the effect of rigidity appears to be the most significant one.

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