SEISMIC-DERIVED ROCK TRUE RESISTIVITY (R_t) REVISITED. PART II: REFORMULATION USING WYLLIE'S TIME-AVERAGE MODEL

by

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I. INTRODUCTION

This paper is the second of a three-part presentation. As highlighted in the previous paper (Part I, Widarsono & Mendrofa, 2006), the main objective of the study is to re-evaluate the potential of acoustic impedance as a source of resistivity data. This essentially came from the very idea of extracting information of resistivity (R_t), data that plays a very important role in the determination of water saturation in reservoir, from seismic-derived acoustic impedance (AI).

As observed in the past view years, there have been a lot of efforts devoted to the extraction of water saturation information from seismic. However, as Widarsono & Mendrofa (2006) put it, most of the efforts were mainly based on pattern recognition activities with little attention was given to the theoretical aspects of relationships between seismic signals and water saturation. The work reported in this three-part presentation is concentrated more as re-establishing (a reformulation of works reported in Widarsono & Saptono, 2003; 2004) the theoretical relationship between resistivity and acoustic impedance.

In the Part I (Widarsono & Mendrofa, 2006), a reformulation between the classical Gassmann acoustic velocity model and shally sand models of Modified Simandoux and Hossin is presented. In the reformulation, a new resistivity function of acoustic impedance has been established. In principle, whenever acoustic impedance data from seismic has been made available resistivity data for the determination of fluid saturation can be estimated.

Despite the theoretical correctness of the resistivity function presented in the Part I, practicallity is not the function's best aspect. In other words, the resistivity function is not an easy one to be used practically. Various parameters (*e.g.* matrix moduli) have to be assumed, since the data cannot easily measured even in the laboratory. This is indeed the main reason why gassmann model, and others such as Biot, has not been used much in day-to-day practices such as log interpretation for porosity determination.

Being aware of such difficulties, in 1954 M.R.J. Wyllie et al proposed their "time average" model (named after its proportional averaging of pore fluid, rock matrix, and shale transit time values to represent transit time of a fluid-filled porous medium) for any practical uses related to P-wave velocity in porous media. Due to its simplicity, the model, as well as its subsequent modifications, has been used extensively since then in some areas especially in log analysis for porosity determination. Considering this simplicity aspect, this three-part study also adopted Wyllie "time average" model into its reformulation works. This Part II paper presents the formulation using Wyllie and the two shally sand models following the same manner that was adopted and presented in the Part I paper.

Summarily, the objectives of the works presented in this paper are:

- To establish a model/method to obtain formation rock true resistivity (R_t) from seismic-derived acoustic impedance (AI),
- To provide correction/modification onto previous works reported in Widarsono & Saptono (2003, 2004), and
- To provide a simpler alternative to the resistivity function yielded from the reformulation works presented in Part I paper (Widarsono & Mendrofa, 2006).

II. WYLLIE "TIME AVERAGE" ACOUSTIC VELOCITY MODEL

It is in theories of acoustic wave propagation in elastic porous media, such as ones proposed by Gassmann, Biot, or Gertsma and Smits that acoustic wave velocities are influenced by the media's porosity. This implies that the theories can be rearranged and used to derive reservoir rock porosity from log data. However, as aforementioned, these models are not sufficiently simple for day to day applications. Accordingly, Wyllie et al (1956, 1958) after numerous laboratory experiment proposed a linear time-average or weighted average relationship between porosity and transit time:

$$\Delta t_p = \phi \Delta t_f + (1-\phi)\Delta t_{ma} \quad (1)$$

or

$$\phi = \frac{\Delta t_p - \Delta t_{ma}}{\Delta t_p - \Delta t_{ma}} \quad (2)$$

for clean and consolidated rocks where ϕ, Δt_p, and Δt_f and Δt_{ma} are porosity, transit time reading, transit time of fluid, and transit time for rock matrix, respectively. Equation (1) can also be expressed in term of velocities by

$$\frac{1}{V_p} = \frac{\phi}{V_f} + \frac{(1-\phi)}{V_{ma}} \quad (3)$$

For porous rocks with shale presence, Equation (1) is expanded into (Dresser Atlas, 1982):

$$\Delta t_p = \phi \Delta t_f + V_{sh}\Delta t_{sh} + (1-\phi-V_{sh})\Delta t_{ma} \quad (4)$$

for shaly sand, where V_{sh} and Dt_{sh} are shale contents and transit time of shale, respectively. For simplification reason, the formulation work using the Wyllie equation only Equation (4) is used to represent acoustic transit time in shaly formations having all types of shale distribution.

For consolidated and compacted sandstones (ϕ < 20%) acoustic transit time is relatively independent of fluid types within the pores (Schlumberger, 1989). However, in some higher porosity sandstones (ϕ > 30%) that have low water saturation (high hydrocarbon saturation) Δt_p may be greater than those in the same formations when water saturated. Similarly, presence of shale is likely to enlarge the Δt_p.

In carbonates that have intergranular porosity the time average model still applies, but in carbonates with secondary porosity (e.g. vugs) the Δt_w is governed mostly by the intergranular porosity. Nevertheless, for large seismic wave amplitudes even large vugs can have effect on the Δt_p.

III. FORMULATION OF RESISTIVITY FUNCTION

As in the case of formulation using the Gassmann model, as presented in the Part I, the formulation using the Wyllie's time average model is started by the acoustic impedance (AI)

$$AI = V_p \rho_b \quad (5)$$

By using the density average equation of

$$\rho_b = \phi\rho_f + V_{sh}\rho_{sh} + (1-\phi-V_{sh})\rho_{ma} \quad (6)$$

then Equation (6) becomes

$$AI = \frac{\rho_b}{\Delta t_p} = \frac{\phi\rho_f + V_{sh}\rho_{sh} + (1-\phi-V_{sh})\rho_{ma}}{\phi\Delta t_p + V_{sh}\Delta t_{sh} + (1-\phi-V_{sh})\Delta t_{ma}} \quad (7)$$

As mentioned earlier, presence of hydrocarbon in the rock with relatively high porosity may affect acoustic wave velocities (i.e. acoustic impedance) significantly. Therefore, the fluid density in the Equation (7) has to be expanded further through the use of $\rho_f = \rho_w + (1-S_w)\rho_{hc}$ into

$$AI = \frac{(\phi(\$_w\rho_w + [1-S_w]\rho_{hc}) + V_{sh}\rho_{sh} + (1-\phi-V_{sh})\rho_{ma})}{\phi(S_w\Delta t_w + [1-S_w]\Delta t_{hc}) + V_{sh}\Delta t_{sh} + (1-\phi-V_{sh})\Delta t_{ma}} \bullet C \quad (8)$$

where S_w, ρ_w, ρ_{hc}, Δt_w, Δt_{hc}, C are water saturation, water density, hydrocarbon density, transit time for water, transit time for hydrocarbon, and a conversion constant, respectively. The conversion constant (0.3048×10^9) is basically a density unit converter from gr/cc into kg/m^3.

By rearranging Equation (8), a water saturation function of

$$S_w = S_{w(AI)} = \frac{C.\phi(\rho_{hc}-\rho_{ma}) + C.V_{sh}(\rho_{sh}-\rho_{ma}) - AI.\phi(\Delta t_{hc}-\Delta t_{ma}) - AI.V_{sh}(\Delta t_{sh}-\Delta t_{ma})}{AI.\phi(\Delta t_w - \Delta t_{hc}) - C.\phi(\rho_w - \rho_{hc})} \quad (9)$$

Through the principle of

$$S_{w(AI)} = S_{w(petrophysics)} = S_w \quad (10)$$

and using Poupon water saturation model (Poupon *et al*, 1954) of

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$$S_w{}^n = \frac{aR_w}{\phi^m}\left[\frac{\left\{\frac{1}{R_t} - \frac{V_{sh}}{R_{sh}}\right\}}{\{1 - V_{sh}\}}\right] \quad (11)$$

to represent $S_{w(petrophysics)}$, Equation (10) becomes

$$R_t = \frac{X}{X.Y + [f(AI)]^n} \quad (12)$$

with

$$X = \frac{aR_w}{\phi^m(1 - V_{sh})}$$

$$Y = \frac{V_{sh}}{R_{sh}}$$, and

$f(AI)$ is the right hand side of Equation (9).

Note that R_t, a, m, and n are rock true resistivity, tortuosity, cementation factor, and saturation exponent, respectively.

In essence, Equation (12) is the targeted resistivity equation. It has been established as a function of acoustic impedance and various acoustic transit time properties. This serves as an alternative to the similar resistivity functions derived from Gassmann model and presented in Part I paper.

IV. DISCUSSION

The resulting resistivity function presented in Equation (12) is essentially simpler than the one formulated using Gassmann model (presented in Part I) despite long in appearance. The various transit time data required by the equation can simply be obtained from log data and from abundant sources of literatures. Unlike in the case of Gassmann – Poupon resistivity function where the various elastic properties needed are in no way to be obtained easily since there is hardly any ready data from log survey and results of past works that are presented in literatures scarcely meet the requirement. It is more often than not that measured elastic properties information are presented in the form of ranges, which is usually of little use for practical purposes.

Although the Wyllie – Poupon resistivity function still needs to prove itself through real trial (presented in Part III) another potential advantage that is associated with this model is that it also use input data that is used by conventional well log analysis. Data such as rock matrix transit time and density are normally known and has been tested for the most representative ones by the log analysts through previous analysis on existing wells supported strong local knowledge.

V. CONCLUSIONS

From the theoretical works presented in this paper, a set of main conclusions have been obtained:

- An alternative theoretical foundation for estimating formation true resistivity data from seismic derived acoustic impedance has been established.
- The formulation of the resistivity function using the Wyllie time average model is far easier than formulation using Gassmann model,
- The Wyllie – Poupon resistivity function is structurally simpler than the Gassmann – Poupon and Gassmann – Hossin models presented in Part I, and therefore is easier to handle.
- The Wyllie – Poupon model, despite its weak theoretical basis when compared to the Gassmann derived models, is potentially more advantageous for practical purposes due to abundant literature sources for assisting selection of input data.
- The use of some input data that is also used in log analysis such as rock matrix density and acoustic transit time will likely to benefit from strong local knowledge in log analysis and petrophysics.

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