DEVELOPMENT OF A SUPPLEMENTARY TECHNIQUE FOR DETERMINING *IN SITU* STRESS MAGNITUDE USING ACOUSTIC WAVE PROPAGATION

by
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I. INTRODUCTION

In accordance with the increasing awareness of the importance of *in situ* stress information in the design of various geotechnical and other petroleum-related subsurface engineering in Indonesia, a complete knowledge of the *in situ* stress is a fundamental requirement. Basically, complete information of the *in situ* state of stress means both the trends and magnitudes of the principal *in situ* stresses. Some stress determination techniques can provide a complete stress tensor (e.g. differential strain analysis, DSA, method), some provide an incomplete tensor (e.g. sleeve fracturing method), and some provide merely the directions of the principal stresses. The Shear wave (S-wave) splitting technique presented by Widarsono et al (1998), following the earlier works made by Yale and Sprunt (1989), obviously falls in the last category. In some cases, which usually do not require *in situ* stress information regarding the magnitudes as an input parameter, principal stress directions still provide useful information. Nevertheless, the expanding use of *in situ* stress information requires, as stated above, a complete information, which means the stress magnitudes as well as directions. Designs of hydraulic fracturing, wellbore stability, and prevention of sand problems are among examples for which information about *in situ* stresses is required.

In relation to the requirement outlined above, the effort which results are presented in this paper was devoted to presenting efforts to predict *in situ* stress magnitude by using ultrasonic wave propagation. This paper mainly presents efforts to find relations between acoustic propagation and *in situ* stress magnitude with an ultimate goal to provide the S-wave splitting technique presented in Widarsono et al (1998) with a means for estimating stress magnitudes.

II. FUNDAMENTAL APPROACH

In differential strain analysis (DSA) method, as presented in Widarsono et al (1998), one of the fundamental assumption for estimating the magnitude of *in situ* stress is in considering the ratios of principal stress-relief microcrack (microcracks that are generated in core samples during their retrieval from reservoir to the surface) closure strains as proportional to ratios of effective principal stresses (there are three principal stresses at any part of the earth's crust, $s\sigma_1$, $s\sigma_2$, and $s\sigma_3$ and $s\sigma_1 > s\sigma_2 > s\sigma_3$, representing tectonic and/or overburden stresses, see Figure 1). As suggested in earlier studies (e.g. Dunn et al, 1973), discontinuities generated during stress-relief processes take the form of microcracks in accordance with the definition proposed by Simmons and Richter (1976).

A microcrack is defined as: "an opening that occurs in rocks having one or two dimensions much smaller than the third. For flat microcracks, one dimension is much less than the other two and the width to length ratio, defined as crack aspect ratio, must be less than 10^{-2} and is typicaly 10^{-3} to 10^{-5}. The length is typically the order of 100 mðm or less" (Simmons and Richter.1976). There are many factors that can influence the generation of microcracks, including thermoelastic contraction, failure in compression, and rapid stress relief such is in the case of core retrieval from reservoirs.

The presence of stress-relief microcracks in freshly retrieved cores is supported by evidence showing that in most cases stress-relief microcracks are indeed present in the form of intergranular or intercrystalline opening, for which the length is usually limited to part of the length of intergranular contact area (e.g. Kranz, 1983). See Engelder (1993) for a more comprehensive discussions regarding formation and evidence of stress-relief microcracks.

In relation with principal stresses in reservoir, stress-relief microcracks are expected to be preferentially aligned normal to the maximum relaxation (the direction of maximum *in situ* compression, sσ_1, before core retrieval) (see Figure 2). Some evidence has also been reported for stress-relief microcracks in relation with the direction of the maximum compression (i.e. the direction of greatest relaxation after core retrieval). For instance, Engelder and Plumb (1984) attributed anisotropy in changes in compressional wave (P-wave) velocity in some granite and sandstones to stress-relief microcracks preferentially oriented normal to s$\eth_1$. Teufel (1983) also observed the anisotropy of P-wave velocities and correlated it with results of anelastic strain recovery (ASR) measurements. He found that the velocity anisotropy disappeared at confining pressure equivalent to the overburden pressure and concluded that the velocity anisotropy was caused by the presence of stress-relief microcracks. These pieces of evidence imply that, in general, the density of stress-relief microcracks present in core samples is much higher than the density of other kind of microcracks that are also probably present.

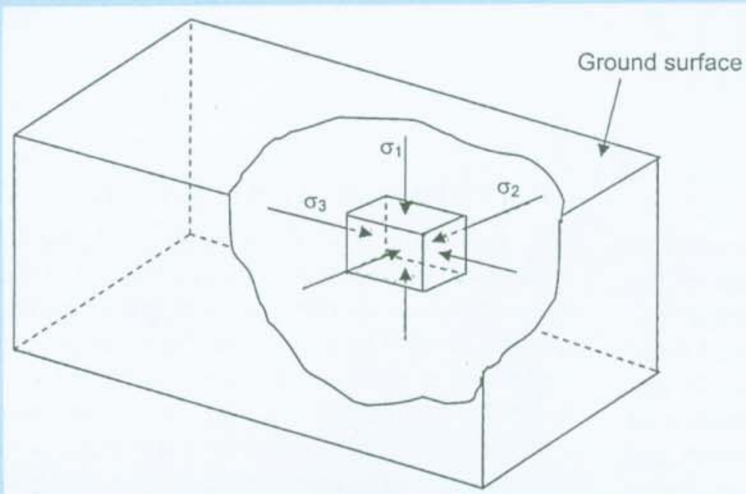

Figure 1
Principal *in situ* stresses with $\sigma_1 > \sigma_2 > \sigma_3$. The σ_1, σ_2, and σ_3, which are perpendicular to each others could represent overburden, tectonic, thermal contraction, or any other sources of stresses depending upon geological setting

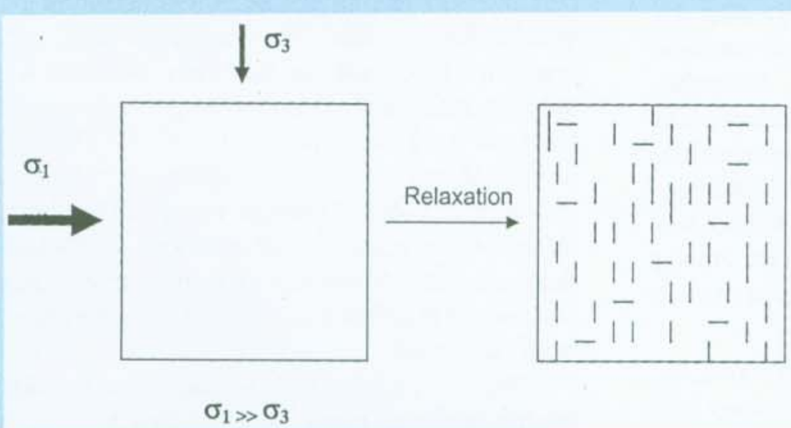

Figure 2
Simple illustration of stress-relief microcracks generated during core retrieval in wellbore, in which stress relaxation occurs. Microcracks with the highest population (i.e. highest density) occur with their opening in the direction of the largest principal *in situ* stress (σ_1) (i.e. the largest relaxation) and orientation in the direction of the smallest principal *in situ* stress (σ_3)

Considering the investigations stated above, it can be assumed with justification that the density of stress-relief microcrack is proportional to the effective *in situ* stress relaxation in the same principal direction. It is a matter of fact that relationship between microcrack density and their closure strain have yet to be found in order to use the assumption of closure strain – *in situ* stress proportionality as used in the DSA. However, with absence of such correlation, and with the fact that acoustic velocities are considerably affected by the presence of

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stress-relief microcracks, it is the main assumption that directional microcrack density is proportional to *in situ* stress relaxation, hence proportional to *in situ* principal effective stress. This serves as the fundamental assumption which leads to the use of Garbin-Knopoff theory of crack-induced effective medium to be described later in this paper for the estimation of microcrack density, hence the magnitude of principal *in situ* stresses.

III. GENERAL ASSUMPTION

Based on discussion above, and also partly on the experience in utilizing S-wave splitting technique to predict *in situ* stress orientations (as presented in Widarsono et al, 1998), the following are some assumptions to be used in the establishment of a stress magnitude estimation technique:

1. Any microcracks present in rock sample are the result of *in situ* relaxation processes.
2. The density of a set of preferentially oriented stress-relief microcracksis proportional to the **effective** principal *in situ* stress in the direction normal to the plane containing them.
3. Any non stress-relief type microcracks possibly present have to be considered as a minimum, randomly oriented and distributed, and which do not significantly affect the ratios of the preferentially oriented stress-relief microcracks.
4. In the absence of any information regarding horizontal stress magnitudes, the overburden stress is assumed as a principal stress in the vertical direction (σ_V), and the horizontal principal stresses are factored by it.
5. Effective modulus theory used in the analysis to obtain microcrack density is valid.

A. *Theory Of Crack-induced Effective Medium*

The presence of any discontinuities, whether stress-relief microcracks or others, in rock medium influences acoustic propagation in the medium due to the fact that the discontinuities change the effective elastic properties that govern the acoustic wave propagation. A number of theoretical works have been devoted to improve the understanding of what happens in a medium containing discontinuities when acoustic energy is transmitted through it. The theoretical works can be classified into two categories, static and dynamic.

In the static approach, modifications of elastic strain energy are carried out by taking into acount the approximation of the energy lost through the presence of the discontinuities. The dynamic approach considers the acoustic scattering of a single inclusion embedded in a solid material and assumes, after summation of the scattering effect by a number of inclusions, a homogeneous effective medium. Some example of works based on the static approach are theories published by Eshelby (1957), Wu (1966), Budiansky and O'Connell (1976), and Nishizawa (1982), whereas Kuster and Toksoz (1974), Garbin and Knopoff (1973, 1975), and Hudson (1980, 1981) represent a selection of those based on dynamic approach. Due to its relative simplicity, the theory of Garbin and Knopoff is chosen as the fundamental theory to be used in estimation of microcrack density.

The theory of effective modulus proposed by Garbin and Knopoff (1973) is based on a solution proposed by Mal et al. (1968) for elastic wave diffraction by a circular disc. In Garbin and Knopof (1973), this solution is extended to involve the solution of the problem of the scattering of long-wavelength compressional wave incident obliquely on a single circular crack embedded in an infinite, elastic, homogeneous, and isotropic medium. This solution can be utilized for determining the effective properties of a medium containing homogeneously distributed cracks (Figure 3). The effective moduli are calculated by considering the composite material as an equivalent homogeneous isotropic elastic material subjected to a constant traction on the surface. It is assumed the crack density is small (i.e. no energy interactions between cracks), so that the lowest total energy approximation of the composite medium is equal to the energy of uncracked matrix plus a correction due to the cracks,

$$E = E_0 + \Delta E = \frac{1}{2}\int_v \sigma^0{}_{ij}\varepsilon_{ij}dv + \frac{1}{2}\int_v \left(\sigma_{ij}\varepsilon_{ij} - \sigma^0{}_{ij}\varepsilon^0{}_{ij}\right)dv \quad (1)$$

where σ_{ij} and ε_{ij} are stress and strain, respectively, and v is the total volume. The first integral is evidently the result in uncracked case, whereas the second is correction due to crack presence. By relating Eq. (1) and the solution for wave scattering by a single coin-shaped crack, Garbin and Knopoff were able to derive an expression for a medium containing a dilute concentration of dry cracks as (Garbin and

Knopoff, 1973):

$$\frac{1}{(\lambda^* + 2\mu^*)} = \frac{1}{(\lambda_0 + 2\mu_0)}\left(1 + \frac{8}{3}\xi\left[\frac{8\mu_0 \sin^2\theta\cos^2\theta}{3\lambda_0 + 4\mu_0} + \frac{(\lambda_0 + 2\mu_0\cos^2\theta)^2}{2\mu_0(\lambda_0 + \mu_0)}\right]\right) \quad (2)$$

where θ is the angle of wave propagation incidence relative to crack normal. Parameters λ and μ are Lame constants with superscripts and subscripts * and 0 denote composite and uncracked medium, respectively. The parameter ξ is the crack density.

For S-waves, Garbin and Knopoff also derived an expression in the same way as for P-waves as follows (Garbin and Knopoff, 1975, after simplification by Crampin, 1978):

$$\frac{1}{\mu^*} = \frac{1}{\lambda_0}\left(1 + \frac{16}{3}\xi\left[\frac{\cos^2\theta\cos^2\psi}{\omega+1} + \frac{(1-2\cos^2\theta)^2}{\omega+1}\sin^2\psi + \frac{(\cos^2\theta\cos^4\theta)}{\omega}\sin^2\psi\right]\right) \quad (3)$$

where ψ is the angle of S-wave polarization relative to the normal of the plane of incidence (e.g. for a wave propagated at $\theta = 90^o$, $\psi = 0^o$ represent an S-wave polarized parallel to the crack plane). Parameter ω is a product of elastic constants of the uncracked medium, $\omega = 2\text{-}(2\mu_0/l\mu_0+2m\mu_0)$. The crack density, ξ, can be determined using Eqs. (2) and (3) if θ and ψ of waves incidence and elastic constants of the uncracked and composite medium are known through equations:

$$V_S = \sqrt{\frac{\mu^*}{\rho_b}} \quad (4)$$

for the S-wave velocity (V_S), and

$$V_P = \sqrt{\frac{\lambda^* + 2\mu^*}{\rho_b}} \quad (5)$$

for the P-wave velocity (V_P). Note that rð$_b$ is density (mass/volume) of the composite medium. When V_P and V_S are known from laboratory measurement, such as the S-wave splitting measurements presented in Widarsono et al. (1998), crack density (ξ) for the three directions of the three principal *in situ* stresses (determined beforehand through S-wave splitting measurements) can be determined using Eqs. (2) through (5).

B. *Measurement Data*

In Widarsono et al. (1998), results of a series of measurements were reported. The measurements, which were performed on some sandstone samples taken from a gas field in the North Sea, included results of DSA and S-wave splitting analysis.

The results of DSA tests include information regarding orientations and magnitudes of the principal *in situ* stresses, which also implied the information regarding the orientations of the stress-relief microcrack sets.

The results of S-wave splitting tests provide information regarding the principal *in situ* stresses directions only since its main working principal is to detect the lowest transmitted S-wave velocities around core samples, at which an incident S-wave experiences its lowest velocity when its direction of polarization takes place in a direction perpendicular to the largest microcrack density (Figure 4). See Yale and Sprunt (1989) for a complete discussion about S-wave splitting technique for detecting direction of principal *in situ* stress directions.

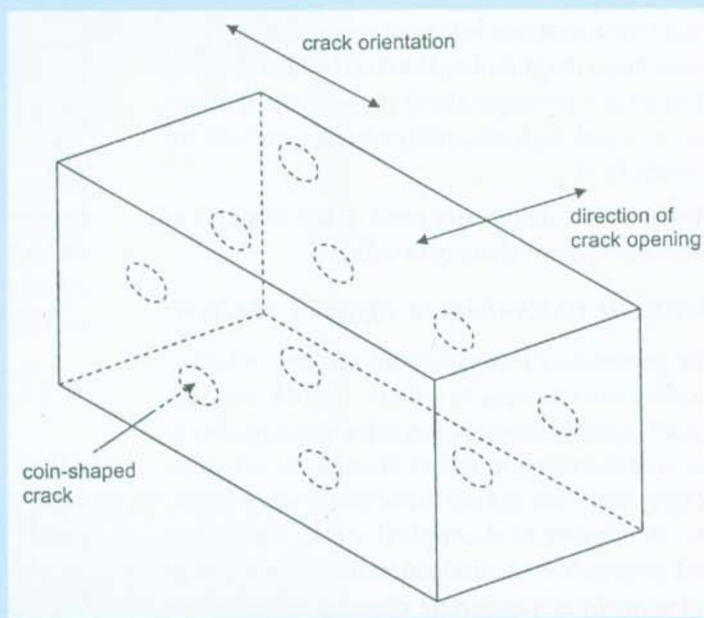

Figure 3
The composite medium on which Garbin and Knopoff is based on

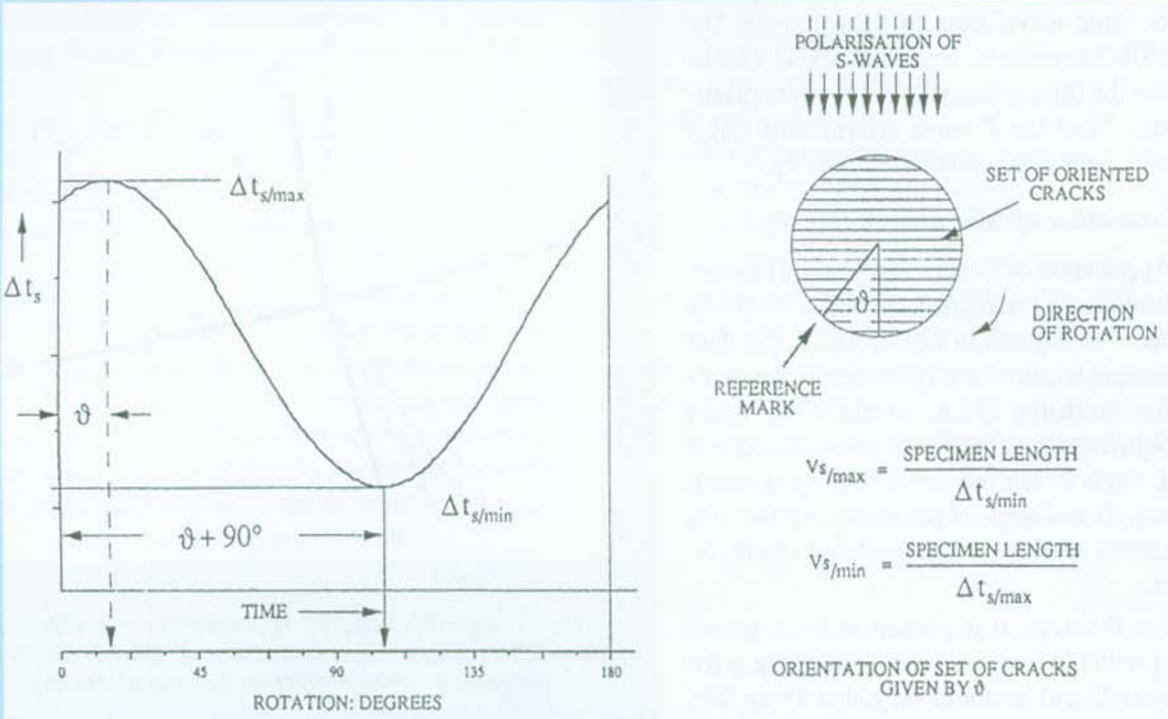

Figure 4
Effect of crack orientation on S-wave as a core sample containing a set of oriented cracks is rotated while an incidence of S-wave is transmitted through it. The S-wave reaches its lowest velocity when its polarization falls in the direction normal to the crack orientation. Note also the conversion from transit time ($\Delta\tau_s$) to S-wave velocity (V_s)

Information about the magnitudes of the stresses is required.

In Widarsono et al. (1998), results of the more established and proven technique of differential strain analysis (DSA) were regarded as the source of comparison for the newer S-wave splitting technique. *In situ* stress orintations produced by DSA were always taken as reference. For the works of testing the technique for determining *in situ* stress magnitude presented in this paper, data from Widarsono et al. (1998) is adopted and results from DSA are adopted as reference for the produced estimates.

Since the DSA results are taken as reference, samples that are considered as the most suitable sample(s) for the requirement of the current work must show similar DSA and S-wave splitting results. Although in general results of the two techniques show similiraty but only one sample (coded SSB–2 in Widarsono et al, 1998), for which DSA and S-wave splitting results indicated sub-vertical direction for σ_1, 88° N and 75° N for σ_2, and 178° N and 165° N for σ_3, respectively. Illustratively, directions of the three principal stresses are shown in Figure 5.

The S-wave splitting data for the SSB-2 sample was also accompanied with data that was fortunately suitable for the current work. The data is a set of V_P and V_S vs. confining pressure at directions of the three principal *in situ* stresses. Note that the data was obtained through true triaxial acoustic wave propagation tests on cubic samples taken from positions adjacent to positions from which the cyllindrical samples for S-wave splitting tests were taken. See Shakeel and King (1998) for a description about the triaxial equipment.

For the application of Garbin and Knopoff not all of the V_P and V_S vs. confining pressure data is needed. Only V_P and V_S at near atmospheric confining pressure (when all microcracks are fully opened) and maximum confining pressure (when all microcracks are fully closed) are needed. They, respectively, represent V_P and V_S for cracked (composite) and uncracked medium. Figure 6 shows three waveforms of measurement for the SSB-2 sample at near atmosperic condition for the three principal *in situ* stress orientations. Note the P-wave arrival time ($\Delta\tau_P$) that was later easily converted into V_P.

C. *Estimation of Microcrack Density*

Application of Garbin and Knopoff theory for determining microcrack density is relatively simple. As implied in Eqs. 2 and 3, the data parameters required are velocities of the composite medium (ði.e. containing open discontinuities), velocities of background material, angle of ray incidence relative to crack normal (θ), and angle of polarization relative to the normal of the plane of incidence (ψ) for S-waves.

For P-waves, θ is chosen as 0° in accordance with the measurement data, whereas for S-waves θ and ψ could be either 0° or 90°. The microcrack density is determined by inversion of Eqs. 2 and 3, depending on the wave mode under consideration. Microcrack densities (ξ) obtained from P-waves are plotted in Figure 7 for the three principal *in situ* stress directions of sample SSB-2. As can be clearly seen, microcrack densities are reduced as more microcracks are closed with the increase of confining pressure.

Microcrack densities obtained from S-waves are generally lower (not plotted) than those obtained from P-waves. However, since V_S values are likely to suffer more averaging errors than V_P during measurements in laboratory, it is considered unneccessary to plot the V_S-derived ξ in Figure 7.

D. *Estimation Of In Situ Stress Magnitudes*

As outlined earlier, for the purpose of estimating *in situ* stress magnitudes, it is assumed that the ratios of stress-relief microcrack density are proportional to the ratios of effective *in situ* stress ($\sigma_{eff1} : \sigma_{eff2} : \sigma_{eff3}$). For this purpose, only the maximum microcrack density values (at the lowest confining pressure in Figure 7) are required, since they reflect the total

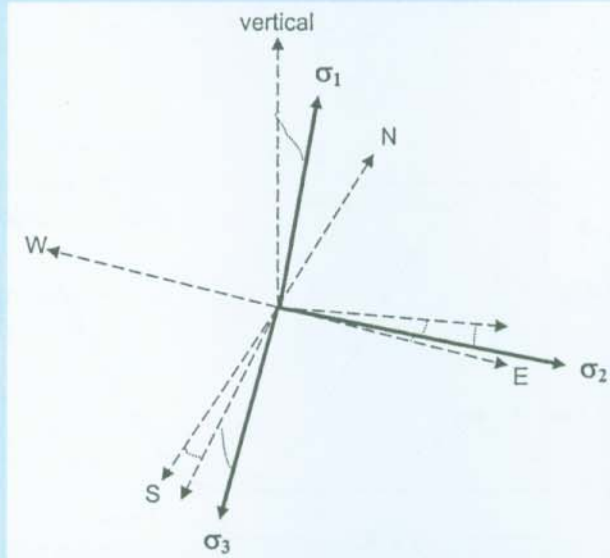

Figure 5
The σ_1 (subvertical), σ_2 (75° N), and σ_3 (165° N) direction for SSB-2 sample determined through S-wave splitting technique. Data from Widarsono et al. (1998)

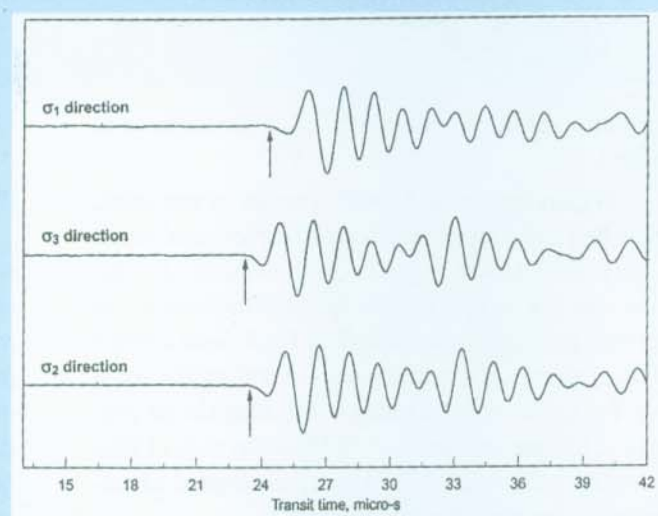

Figure 6
Waveforms picked in the directions of the three principal *in situ* stresses. Arrows indicate P-wave transit time. These directions were previously determined using S-wave splitting technique. Consistently, direction of σ_1 is indicated by the largest transit time (i.e. the slowest waveform) due to its path that is in the direction normal to the orientation of the largest microcrack density. Data from (Widarsono *et al.*, 1998)

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stress relaxations. Additional data are provided by the overburden stress and pore pressure at the point under consideration. The stresses were determined following Terzaghi's equation (as used in the DSA):

$$\sigma = \sigma_{eff.} + P_f \qquad (6)$$

where σ, σ_{eff}, and P_f are *in situ* stress, effective stress, and pore pressure, respectively. When ratios of crack densities ($\xi_{\sigma 1} : \xi_{\sigma 2} : \xi_{\sigma 3}$) in the three principal directions are known, hence the ratios of σ_{eff} values, the horizontal *in situ* streses can be calculated provided the vertical stress (i.e. overburden stress, the only known source of *in situ* stress) and pore pressure are known. In the estimation of stress magnitudes, stress gradient of 0.0237 MPa/m and 0.0102 MPa/m were used to predict the vertical absolute *in situ* stress and pore pressure,respectively. The results of the calculation are listed in Table 1. Note that the σ_1 value for DSA is not exactly equal to the overburden stress, since the technique predicts that σ_1 dips at 16° from the vertical (or 74° from horizontal), hence giving slightly different magnitude of *in situ* stress. In the other hand, the S-wave splitting technique has to assume, at least in this case, that the σ_1 is exactly in vertical direction hence simplifying the problem by the use overbuden stress as one of the principal *in situ* stresses.

The comparison of stress magnitude estimates in Table 1 demonstrates a reasonably good agreement between DSA and the acoustic technique proposed in this paper. If the results from DSA is taken as the reference for comparison the results exhibited by the acoustic method are very encouraging even though differences, especially in the σ_{eff} estimates are still shown . This is true when it is considered that the parameters used for predicting the effective stress ratios are different (DSA employs microcrack closure strains rather than microcrack densities), and it is realized that there is no known relationship between crack density and crack closure strain. Results in Table 1 have demonstrated that the relationship between the two parameters is indeed non-linear, even though the same results have also demonstrated that the parameter of crack density can be utilized for estimating the magnitude of *in situ* stress.

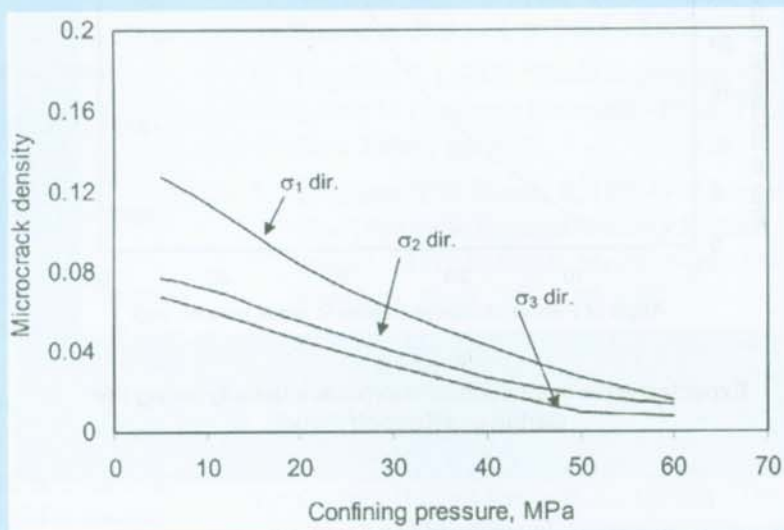

Figure 7
Calculated microcrack density for the three principal directions

Table 1
Comparison of stress magnitude estimates from DSA and the proposed acoustic techniques

Tech.	Microcrack Density			Eff. stress ratio	σ_1	σ_2	σ_3
	σ_1 dir.	σ_2 dir.	σ_3 dir.	$\sigma_{1eff} : \sigma_{2eff} : \sigma_{3eff}$	(MPa)	(MPa)	(MPa)
DSA	-	-	-	1.38 : 1.10 : 1.00	79.1	73.3	69.5
Acoustic	0.127	0.076	0.067	1.88 : 1.14 : 1.00	78.3	60.3	57.4

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Table 2
Comparison of stress magnitude estimates from DSA and the proposed acoustic techniques after correction on microcrack density

Tech.	Microcrack Density			Eff. stress ratio	σ_1	σ_2	σ_3
	σ_1 dir.	σ_2 dir.	σ_3 dir.	$\sigma_{1eff} : \sigma_{2eff} : \sigma_{3eff}$	(MPa)	(MPa)	(MPa)
DSA	-	-	-	1.38 : 1.10 : 1.00	79.1	73.3	69.5
Acoustic	0.119	0.076	0.063	1.88 : 1.21 : 1.00	78.3	62.4	57.4

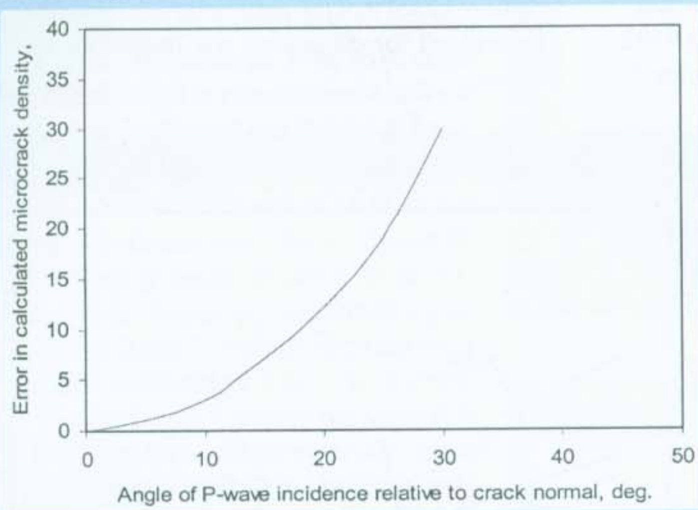

Figure 8
Expected error in calculated microcrack density using the Garbin and Knopoff model

E. *Error in Prediction Due to Deviation of Ray Path from Crack Normal*

Results from rock samples analysed using DSA in both Table 1 and Widarsono et al. (1998) have demonstrated that it is the rule rather than the exception that the planes of the stress-relief microcracks do not neccessarily lie in either horizontal or vertical directions, for reasons of stresses local deviation due to factors such as rock heterogeneities. In determining stress magnitudes from microcrack density, it is neccessary for the cubic rock sample to be tested to have a pair of surfaces perpendicular to the vertical. This is because the overburden stress is the only known stress which is always assumed as the principal stress in the vertical direction (σ_v). Moreover, it is sometimes impossible to prepare the sample specimen with parallel faces in directions dictated by other sources, such as DSA, due to dimensional limitations of the commonly available core samples.

The considerations stated above underline a probable error in estimating microcrack densities. This is true when it is considered that velocities (e.g. P-waves) of energy travelling in a direction normal to the crack plane and in a direction oblique to it are different in magnitude. The error arises when in both cases it is assumed that the ray path is parallel to the crack normal, thus disguising the real crack density required fro estimating stress magnitude.

In order to investigate the error in crack density estimation, a separate investigation has been carried out using the Garbin and Knopoff theory (Eqs. 2 and 3). The data used was also the SSB-2's V_P and V_S at near atmospheric condition data. The investigation was carried out by keeping the two velocities constant while changing the angle of ray incidence from crack normal (θ) from 0^o to 30^o, and recording the change in crack density given by the calculation. It was found that the use of Eqs. 2 and 3 for the three principal directions produced an identical degree of error for both P- and S-waves. The results are plotted in Figure 8.

Figure 8 demonstrates a situation for which an underestimation of crack density could be as high as 30% at a ray path angle of 30^o from the microcrack normal. Ratios of microcrack densities which include a potential error of this level can, undobtedly, be significantly misleading. The case of SSB-2 sample provides an example of this potential underestimation. DSA results for SSB-2 sample used in this paper (Table 1) show that σ_1 and σ_3, and consequently the microcracks, dip from vertical and horizontal, respectively, by 16°. According to the plot in Figure 8, this causes underestimation of approximately 6% in microcrack densities in these particular principal directions. If this error level is applied to the results listed in Table 1, changes are observed over the effective stress ratios (Table 2). After correction, it is obvious that improvement is made for σ_2 (major horizontal *in situ* stress) resulting in a change from 60.3 to 62.4 MPa, which is closer to the magnitude shown by DSA results. Although this correction does not mean that all microcrack densities obtained from rock specimens must be corrected in this manner, but this can be taken as precautions before reaching a final conclusion about the obtained microcrack densities to be used in the estimation of *in situ* stresses.

IV. CONCLUSIONS AND RECOMMENDATIONS

To summarize the works presented in this paper, a set of conclusions have been made:

1. A technique for estimating *in situ* stress magnitude has been proposed. This technique is designed to provide a suplementary support to the S-wave splitting technique that enables the estimation of *in situ* stress orientation only.
2. Density of stress-relief microcracks derived from acoustic measurements can be used for the estimation of *in situ* stresses. This is to be of a special importance when compared to microcrack strain closure that is used by the established differential strain analysis (DSA) technique.
3. Deviations of acoustic propagation from direction of crack normal may cause error as high as 30% in the calculated microcrack density (about 6% in the resulting *in situ* stress magnitude) using the Garbin and Knopoff theory. This must be taken as a precausion when employing this technique in future uses.

Based on the findings resulted from the works presented in this paper, a set of recommendations should be proposed for further refinement of the technique:

1. Further investigations using the proposed technique are needed using more rock samples containing stress-relief microcracks.
2. Application of other crack-induced effective media theories. It is hoped that, with the use of various theories comparisons could be made to find the most reliable theory.

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