A SINGLE PHASE MODEL FOR ANALYZING GAS PIPELINE NETWORKS

By
Edward ML Tobing

ABSTRACT

A single phase flow model has been developed for gas distribution pipeline networks. The model is developed based on looped-system approach with some modifications. In this model, a equation of state model is implemented for predicting the gas properties required for the governing equations of the network system. By utilizing the Linear Theory Method, the Panhandle's single phase gas flow model is implemented in this model to predict the hydrodynamic variables in each leg of the network using the iterative technique which is developed in this study. A generalization of the single phase network model is provided thereby making it possible for the single flow model used to be replaced by another one that may be more applicable for a particular situation. Using the iterative procedure developed, pressure at all nodes, gas flow rate at each leg can be predicted. The test results demonstrate that the model can serve as a predictive and design tool for solving a single phase gas flow problem in pipeline network.

Keyword : A single phase model, gas pipeline networks

I. INTRODUCTION

Natural gas collected from producing wells is often distributed to the consumer via pipeline networks. With the ever increasing use of natural gas, pipeline network are becoming larger and more complex. The design and analysis of these complex pipeline networks often involve extensive calculation. The problem of compressible fluid flow through pipelines and conduit has been studied by many investigator. For pipelines, the most commonly used equations for these calculation are the Weymouth (equation) or the Panhandle equations.

One of common features of gas gathering and distribution system is that they are usually made up of network of a pipelines with complex arrays of loops, nodes, compression stations, etc. In order to optimize such a system, a system analysis approach is called for. This has not been possible mainly because of the process modeling intricacies involved. One aspect of this problem that is almost intractably challenging is the hydrodynamic behavior of a network of pipelines under single phase conditions. Even though the operation of a gas gathering and/or distribution pipeline network is anything but steady, it is not uncommon to assume steady state flow in order to simplify the problem and render it tractable.

The main objective of this work is to attempt a formulation of a network model for gas pipeline network. Single phase approach is used in conjunction with Panhandle's equation serving as the model for estimating hydrodynamic variables in any leg of the network. This variable include pressure drop. This model is coupled to Peng Robinson equation of state based phase behavior model, to render the model single phase. The resulting model is reformulated into a form that expresses pressure loss as function of mass flow rate. The functional form is implicit rather than explicit. It should be recalled that the conventional head loss model used in pipeline network analysis are usually explicit, thus making them amenable to the standard formulation strategies. By invoking continuity equation at each node in the network and energy conservation in each loop, using mass flow rate throughout rather than volumetric flow rate, the

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complete set of network equations are formulated. Solution of these equation is accomplished by using the Linear Theory Method.

II. FORMULATION OF THE MODEL

The model formulated to resolve a single phase gas flow problem in a complex network of pipelines consists of three main components. These are the single phase flow model, the looped network analysis model, and the phase behavior model. The application behavior in this model is mainly to determine the phase properties which are required for the governing equations. The Peng-Robinson equation of state (1976) is used as the basis for the phase behavior model. The model predicts the thermo physical properties of the single phase, for either single-component or multi component systems.

The looped network model is basically developed by adopting Kirchhoff's laws for electrical circuits. Cross developed a looped network model for pipeline system by applying the continuity equations and the energy equations at the node (junction) and the loop, respectively. The energy loss in each leg of the loop is required for the energy balance equation. This is supplied by a formulated Panhandle single phase flow equation.

In formulating the model, four major assumptions are made. They are :

1. Fluid temperature is constant across the system,
2. Steady state conditions,
3. Fluid composition is uniform across the system,
4. Effects of elbows, valves, fitting, or meters are ignored.

A. *The Single phase Flow Model*

In order to predict single phase gas flow behavior in pipeline network, any single phase gas flow model can be implemented in the model by coupling it to the system of equations for the network system. It will be assume that the single phase gas flow model is expressible in the form:

$$\Delta P_i = \gamma_i \, Q_i^{\alpha_i} \tag{1}$$

Equation (1) is the conventional non-linear relationship between pressure drop and flow rate. Each of the term in the energy balance equations for each loop in the network takes this form. Using the Linear Theory Method, equation (1) can be linearized as follows :

$$\Delta P_i = \gamma_i \, Q_i^{(\alpha_i - 1)} \, Q_i \tag{2}$$

Defining,

$$\beta_i = \gamma_i \, Q_i^{(\alpha_i - 1)} \tag{3}$$

By substituting equation (3) into equation (2), the single phase gas flow equation can be expressed as

$$\Delta P_i = \beta_i \, Q_i \tag{4}$$

The pressure drop in equation (4) can also be expressed in terms of mass flow rate, W_g, instead of volumetric gas flow rate, Q_i. This relationship can be written as

(5)

Where i indicates pipeline number.

The single phase gas flow equation can be written as

$$\frac{\Delta P}{\Delta Z} = \frac{g}{g_c} \rho_g \sin\theta + \frac{f \, \rho_g \, v^2}{2 \, g_c \, d} \tag{6}$$

Using algebraic manipulation, equation (6) can be expressed in the same form as equation (5) where h_i, is defined by

$$\eta_i = \frac{1}{\Delta Z \left\{ \frac{C_1}{W_g} + C_2 \, W_g \right\}} \tag{7}$$

The derivation of the expression for C1 and C2 follows. In this model, single phase gas flow model is implemented in the system of equations for a network system to predict single phase flow performances of gas flow in pipelines. In order to implement this model into the network equations, the single phase gas flow equations needs to be rearranged in such a way that the pressure loss is presented in term of the mass flow rate. In this case, the linier theory method is used to linearize the flow equation.

Defining that, $W_g = \rho \, v \, A_p$ (8)

Using equation (8), the gradient pressure shown in equation (6) becomes

(9)

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$$\Delta P = \Delta Z \left\{ C_1 + C_2 \, W_g^{\,2} \right\} \qquad (10)$$

Where

$$C_1 = \frac{g}{g_c} \rho_g \sin\theta \qquad (11)$$

$$C2 = \frac{8 \, f_g}{g_c \, \pi^2 \, d^5 \, \rho_g} \qquad (12)$$

Friction factor gas, $f_{g,}$ can using Panhandle-A equation

$$f_g = \frac{0.085}{N_{\mathrm{Re}}^{\,0.147}} \qquad (13)$$

or using Panhandle-B equation

$$f_g = \frac{0.015}{N_{\mathrm{Re}}^{\,0.0392}} \qquad (14)$$

where

$$N_{\mathrm{Re}} = \frac{20 \, W_g \, \gamma_g}{\rho_g \, B_g \, \mu_g \, D} \qquad (15)$$

Using the Linear Theory Method, the pressure gradient equation becomes

$$\Delta P^{n+1} = \Delta Z \left(\frac{C_1}{W_g^{\,n}} + C_2 \, W_g^{\,n} \right) W_g^{n+1} \qquad (16)$$

or

$$\Delta P^{n+1} = \eta \, W_g^{n+1} \qquad (17)$$

or

$$W_g^{n+1} = \eta \, \Delta P^{n+1} \qquad (18)$$

For the i-th leg, the equation is written as

$$W_{gi}^{n+1} = \eta_i \, \Delta P_i^{n+1} \qquad (19)$$

B. *The Pipeline Network Model*

In order to properly set up the system of equations for a network system using this model, certain rules and conventions must be defined. These conventions are expressed below:

1. In each loop, the assigned flow direction in all the legs must be the same, clock-wise or counter clock wise. Initially (i.e. at the beginning of iterative process), the flow is assumed to be positive in the assigned direction. Of course, before the termination of the iterative process, each flow will assume the appropriate sign to satisfy continuity at each node and energy balance in each loop.
2. The flow in a leg common to two loops must have the same flow direction.
3. The flow leaving from a node is assigned a negative sign, while the flow coming to the node is given a positive sign.
4. Each node must be specified by an integer number. This number is completely arbitrary with the condition that numbering starts from 1 and increase sequentially.
5. Similarly, each loop must be specified by an integer number arbitrarily. However, the number used in the system must start from 1 and increase sequentially.

Figure 1 shows an example of a looped pipeline network system consists of 4 nodes and 2 loops.

C. *The Continuity Equations*

By adopting Kirchhoff's laws for any junctions in a closed circuit system, the continuity equation can be applied for each node and expressed as

$$\sum_{j=1}^{N_i} (\rho \, v \, A_p)_{ij} = 0 \qquad ; i = 1,\ldots,N \qquad (20)$$

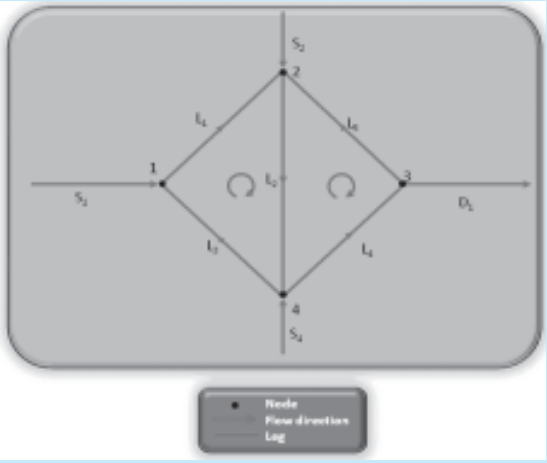

Figure 1
The example of a looped pipeline network system

or, in general, equation (19) can be written as

$$\sum_{j=1}^{N_i} (W_g)_{ij} = 0 \qquad ; i = 1,\ldots, N \qquad (21)$$

In the presence of the mass flow rate coming from an external source or supply and leaving from the system or demand, the continuity equation can be written as

$$\sum_{j=1}^{N_i} (W_g)_{ij} = D_i - S_i \qquad ; i = 1,\ldots,N \qquad (22)$$

Where

i = i-th node within the network

j = j-th leg connected to node i

N_i = number of legs connected to node i

N = total number of nodes within the network

D_i = demand or flow leaving the system from the i-th node

S_i = supply or flow coming into the system through node i

D. *The Energy Equations*

By adopting the Kirchhoff's loop law for any closed circuit, the energy equations for the looped network system can be written in the following form:

$$\sum_{k=1}^{N_l} \Delta P_{kl} = 0 \quad ; l = 1,\ldots, M \quad (23)$$

Where

kl = k-th leg in the l-th loop

l = l –th loop within the network

N_l = number of legs within the l-th loop

M = total number of loops within the network

The system of equations for the pipe network, therefore, consists of both continuity and energy balance, which are expressed by equations (22) and (23), repectively. With the network made of N nodes, we can write N continuity equations. However, only N-1 of these equations are linearly independent, hence only these many are admissible. For each of the M loops, one energy balance equation can be written giving M non-linier independent equations. It can be proved that a pipe network made up of N nodes, M non-overlaooing loops and L legs will satisfy the equation L=(N-1)+M. This means that M = (L – N + 1). Thus, we can write (L – N +1) energy equations. Therefore, the total number of equations we have is (N-1) + (L-N+1) or L equations. Since the unknown are the L flow rates in the legs (from which pressure can be calculated by using equations (5), we have a complete set of equations which can be solved for these flow rates. By considering a pipeline network system as shown in Figure 2, the following is an example of the system of equations generated for that system using the above system of equations (21) and (22).

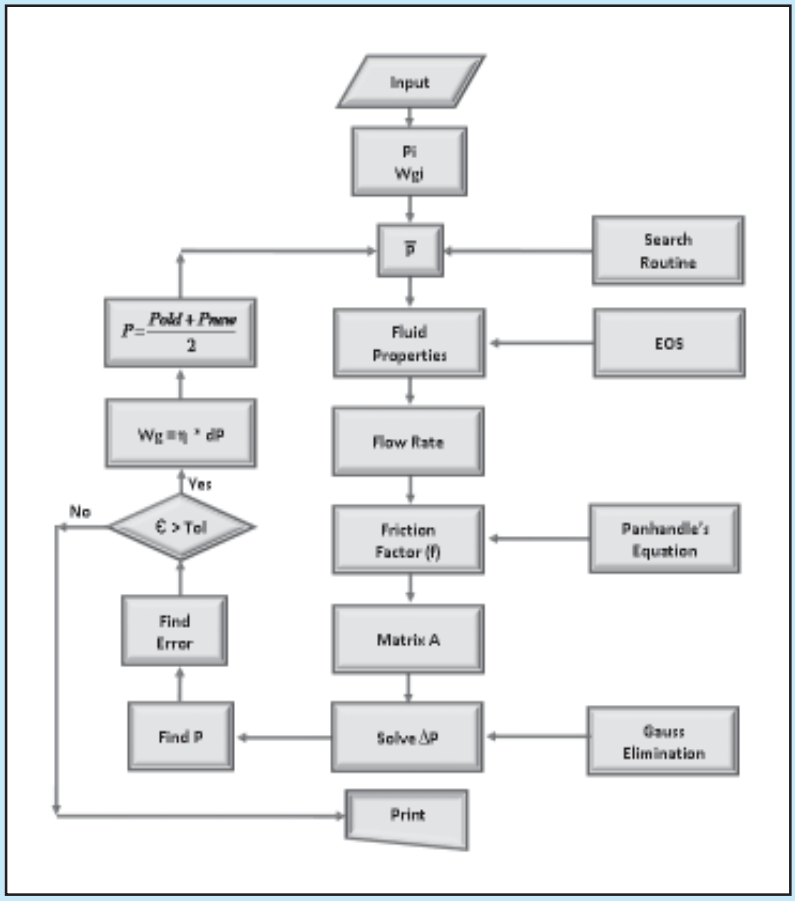

Figure 2
The Flow chart of the calculation procedure

$$
\begin{array}{lllllll}
- W_{g1} & +0 & + W_{g3} & +0 & +0 & = - S_1 \\
+ W_{g1} & - W_{g2} & +0 & +0 & + W_{g5} & = - S_2 \\
+0 & +0 & +0 & + W_{g4} & - W_{g5} & = +D \\
+\Delta P_1 & + \Delta P_2 & + \Delta P_3 & +0 & +0 & = 0 \\
+0 & + \Delta P_2 & +0 & + \Delta P_4 & + \Delta P_5 & = 0
\end{array}
$$

By subtituting equation (5) into equation (22), the following system equations representing continuity at each node "i" can be written in general form as

$$\sum_{j=1}^{N_i} A_{ij}\, \Delta P_j = R_i \qquad ; i = 1,\ldots, (N-1) \qquad (24)$$

Where

$$A_{ij} = B_{ij}\, \eta_j \qquad ; i = 1,\ldots, (N-1) \qquad (25)$$

$$R_i = D_i - S_i \qquad ; i = 1,\ldots, (N-1) \qquad (26)$$

For the system of equations representing energy equations at each loop "l", on the other hand, the equations can be written in general form as

$$\sum_{j=1}^{N_i} A_{lj}\, \Delta P_j = R_l \qquad ; l = 1,\ldots, M \qquad (27)$$

Where $A_{lj} = B_{lj} \qquad ; l = 1,\ldots, M \qquad (28)$

And $R_l = 0 \qquad ; l = 1,\ldots, M \qquad (29)$

ij = refer to the j-th leg connected to the i-th node
L = total number of legs in the network
N = total number of nodes in the network

Equation (23) and (26) show that the set of pressure drops in the leg, (ΔP_j), constitutes the dependent variable vector for the system of equations describing flow in the network.

Using equation (23) and (26), the system of equations generated for the network system in Figure 2 becomes

$$
\begin{array}{lllllll}
- \eta_1 \Delta P_1 & +0 & + \eta_3 \Delta P_3 & +0 & +0 & = - S_1 \\
+ \eta_1 \Delta P_1 & - \eta_2 \Delta P_2 & +0 & +0 & + \eta_5 \Delta P_5 & = - S_2 \\
+0 & +0 & +0 & + \eta_4 \Delta P_4 & - \eta_5 \Delta P_5 & = + D_3 \\
+ \Delta P_1 & + \Delta P_2 & + \Delta P_3 & +0 & +0 & = 0 \\
+0 & + \Delta P_2 & +0 & + \Delta P_4 & + \Delta P_5 & = 0
\end{array}
$$

This system of equations can be written in the matrix form as

$$
\begin{bmatrix}
- \eta_1 & 0 & \eta_1 & 0 & 0 \\
\eta_1 & - \eta_1 & 0 & 0 & \eta_1 \\
0 & 0 & 0 & \eta_4 & -\eta_5 \\
1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \end{bmatrix}
=
\begin{bmatrix} -S_1 \\ -S_2 \\ +D_3 \\ 0 \\ 0 \end{bmatrix}
$$

In more compact notation, this can be written as

$$[A] \quad \vec{\Delta P} = \vec{R} \qquad (30)$$

Where A is an L x L square matrix and L is total number of legs in the network system.

Using equation (29), the pressure drop in each leg of the network can be solved using any matrix technique. A complication arises from the fact that some element of the coefficient matrix are not constants but functions of W in each leg, which in turn is a function of the (ΔP_j), the dependent variable vector. Since the mass flow rate in each leg is also an unknown variable, an iterative procedure is required in this calculation. In this model, the mass flow rate, W is calculated iterative using equation (5). The iteration equation is formulated as

$$(W_g)_j^{(K+1)} = \eta_j^{(K)}\, \Delta P_j^{(K+1)} \qquad (31)$$

In this case, ΔP is first predicted using the old value of mass flow rate.

E. *The Generalized Pipeline Network equation*

Suppose N and L are the number of nodes and the number of legs in a network system, respectively, the system of equations will consist of (N-1) continuity equations and (L-N+1) energy equations. In general, the continuity equations can be expressed by

$$\sum_{j=1}^{L} B_{ij}\, \delta_j\, W_{gj} = D_j - S_j \qquad ; i = 1,\ldots,N-1 \qquad (32)$$

And the energy equations are expressed by

$$\sum_{j=1}^{L} B_{ij}\, \delta_j\, \Delta P_j = D_j - S_j \qquad ; i = N,\ldots L \qquad (33)$$

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where

$$\delta j = \frac{W_{g\,j}}{|W_{g\,j}|}$$

$$B_{ij} = \begin{cases} +1 \\ 0 \\ -1 \end{cases}$$

The value of B is equal to +1 when the flow is leaving from the node, to 0 when there is no pipe attached to the node, and to -1 when the flow is coming to the node. For energy equations, the value of B is positive when the flow is in the same direction as the assigned flow at the beginning, otherwise, the flow direction is opposite to the assigned flow. By substituting equation (18) into equation (31), the system of continuity equations becomes

$$\sum_{j=1}^{L} B_{ij}\,\delta_j\,\eta_j\,\Delta P_j = D_i - S_i \quad ; i = 1,\ldots., N\text{-}1 \quad (34)$$

Therefore, the system of equations for both continuity and energy equations can be expressed by

$$\sum_{j=1}^{L} A_{ij}\,\Delta P_j = R_i \quad ; i = 1,\ldots., L \quad (35)$$

where

$$Aij = \begin{cases} B_{ij}\ \delta_j\ \eta_j & \text{for i = 1,.....,N-1} \\ B_{ij}\ \delta_j & \text{for i = N,..... ,L} \end{cases} \quad (36)$$

$$Ri = \begin{cases} Di - Si & \text{for i = 1,.....,N-1} \\ 0 & \text{for i = N,..... ,L} \end{cases} \quad (37)$$

In matrix form, equation (34) can be written as

$$[A]\ \vec{\Delta P} = \vec{R} \quad (38)$$

Using Gaussian Elimination technique, $\vec{\Delta P}$ can be obtained. Since the value of W_g and P are also unknown, an iterative procedure is then required to resolve this problem. An iterative procedure has been developed in this study to solve both the pressure loss, ΔP and mass flow rate, W_g, in each leg of the pipeline network. In this iterative process, the convergence must be achieved in both mass flow rate and pressure loss.

F. *The Phase Behavior Model*

The application of the phase behavior model is to determine single phase properties which are required in order to render the governing equation complete. In this model, Peng-Robinson EOS's is basically used to estimate the single phase gas properties in each leg of the system when the average pressure changes due to the pressure drop. Since temperature is assumed to be constant, the phase properties are dependent on the pressure in each leg.

III. NETWORK IDENTIFICATION

The first part is related to input data handling strategy. This is necessary to properly identify the pipeline network system. Since the size and configuration of the pipeline network in field vary from place to place and there is no specific rule for identifying the network, it is important to design the code in such a way as to facilitate use. In addition, the code also has to be able to handle all information given by the user in the input mode and evaluate them properly.

The identification of the pipeline network could be very complicated when the numbering used for identifying the network is not in order. This problem usually arises when the network under consideration is very large and complex, which is often the case. In this model, a subroutine has been developed to resolve the problem where the identification number used by user can be located in an arbitrary position in the network. In order to execute this routine, the data needed to be obtained from the input mode are:

- number of legs connected to each node in the network
- leg number and its flow direction at each node
- number of legs within each loop
- leg number within each loop
- node number within each loop.

Using these pieces of information, network identification can be accomplished and the system of equations (36) can be generated by the following procedure and algorithms

1. Using the information obtained from the input mode, first find the coefficients of matrix B as defined in equation (37)

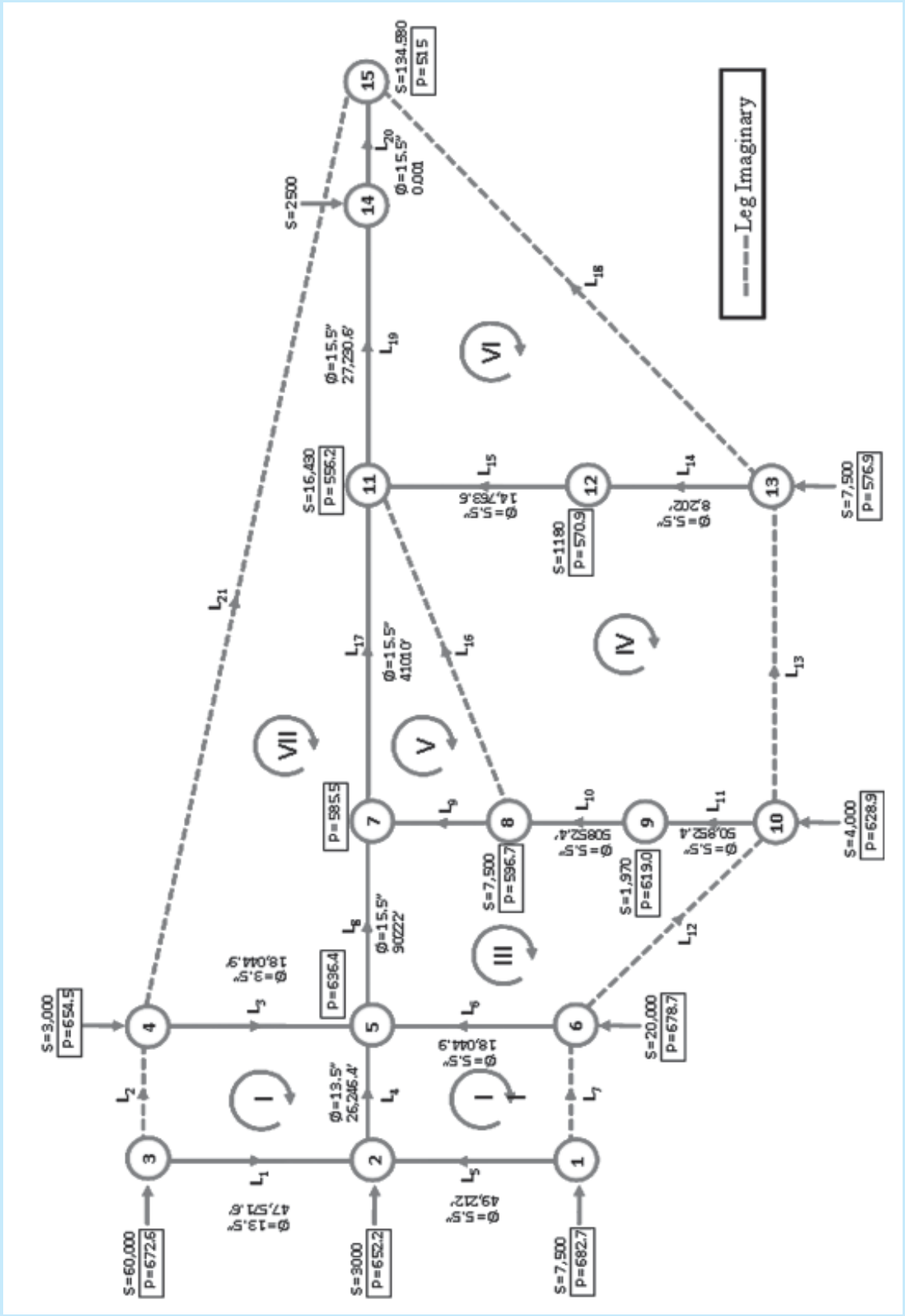

Figure 3
Predicted pressure distribution for example case

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2. Using the hydrodynamic and gas properties of the system, find the coefficient factor of each leg, η
3. Compute the elements of matrix A as defined in equation (30).

Using the above algorithms, the coefficients of matrix A can be determined using the information obtained from the input mode. This algorithm is quite simple and general. It is capable of handling arbitrary identification of the elements of the network. The iteration procedure is detailed out in a flow chart (see Figure 2).

IV. EXAMPLE ANALYZING GAS PIPELINE NETWORKS

In this study, the model was tested using various hypothetical data of pipe line network and using friction factor Panhandle-A equation. For this case, the network consists of 21 pipe legs, 15 nodes, and 7 loops. The fluid data used for testing this model consists of specific gravity (0.697) and temperature (540 R).

The input data used to test this model include :

- (S)upply or flow rate coming to the system (Mscf/D)
- (D)emand or flow rate leaving the system (Mscf/D)
- Pipeline length (ft)
- Pipeline diameter (in)
- (P)ressure (psi)
- Temperature (R)
- Elevation (ft)

The results of the test example which are presented in Figure 3 and Table 1 include :

- (S)upply or flow rate coming to the system (Mscf/D)
- (D)emand or flow rate leaving the system (Mscf/D)
- Pipeline length (ft)
- Pipeline diameter (in)
- (P)ressure at each node (psi)
- Gas flow rate in each leg (Mscf/D)

From the test, it is also found that number of iterations required to converge. The iteration required to converge is 26.

V. CONCLUSIONS

a. The results of prediction obtained from this test shows that the model is capable of predicting pressure distribution at each node and gas flow rate in each leg for a pipeline network.

b. Data input handling is simple and the results are also displayed in a way that make it attractive to engineers involved in analyzing pipeline networks.

NOMENCLATURE

A = Square matrix of the system of equations

A_p = Cross-sectional area of pipe, ft^2

LEG	LENGTH (feet)	D, PIPE (inch, ID)	Qgas (Mscf/D)	dP (psi)
L_1	47.571,6	11,6	60.000,0	20,3
L_2 *	0,0	0,0	0,0	0,0
L_3	18.044,0	3,5	3.000,0	18,1
L_4	26.246,4	13,5	70.500,0	15,9
L_5	49.212,0	5,5	7.500,0	30,4
L_6	9.842,4	5,5	20.000,0	42,3
L_7 *	0,0	0,0	0,0	0,0
L_8	90.222,0	15,5	93.500,0	50,8
L_9 *	0,0	0,0	0,0	0,0
L_{10}	50.852,4	5,5	6.000,0	22,3
L_{11}	50.852,4	5,5	4.000,0	9,9
L_{12} *	0,0	0,0	0,0	0,0
L_{13} *	0,0	0,0	0,0	0,0
L_{14}	8.202,0	5,5	7.500,0	6,0
L_{15}	14.763,6	5,5	8.680,0	14,7
L_{16}	82.020,0	7,5	13.470,0	40,5
L_{17}	41.010,0	15,5	93.500,0	29,3
L_{18} *	0,0	0,0	0,0	0,0
L_{19}	2.723,6	15,5	132.080,0	41,2
L_{20}	0,0	15,5	134.580,0	0,0
L_{21} *	0,0	0,0	0,0	0,0

*: Leg imaginary

B = Matrix coefficients whose values are +1 or -1 or 0
D = Demand, lbm/D
D = inside pipe diameter, in
F = pipe friction factor
g = Gravity acceleration, ft/s^2
g_c = Conversion factor
k = time level in the iteration calculation
L = Total number of legs
N = Total number of nodes
M = Total number of loop in a network
R = Right hand side of the system of equations
P = Pressure, psi
Q_g = Volumetric gas flow rate, cuft/D
S = Supply, lbm/D
T = Temperature, Rankine
W_g = Mass flow rate of gas, lbm/D
Z = Length or distance, ft
f_g = friction factor gas
μ = viscosity, cp
η = Single flow equation coefficient
ρ = density, lbm/ft^3
ε = tolerance

Subscripts

g = gas phase

Superscripts

k = Time level
$\bar{}$ = Vector

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